Compiler-Generated Frame Accessors in Exact WebAssembly Proofs

Anonymous manuscript

10 August 2026

Abstract

Direct proofs about decoded WebAssembly must establish how compiler-created local frames encode mathematical loop state. LeanExe now emits Lean theorems for the parameter list, local-list length, operand stack, and every combined local getter of each recognized array-fold continuation frame. Its proof recipe places these theorems beside two shared result-frame getter theorems in a structured library retrieved by a coding agent. A manual fixed-artifact substitution removed ten proof-local projection declarations and reduced proof source from 588 to 555 lines. Three fresh agents then constructed exact-artifact proofs for bounded addition, multiplication, and bitwise-XOR folds, and a separate verifier accepted all three. The proof stages took 2231, 2099, and 2452 seconds. The addition and multiplication runs were slower than their fixed-artifact predecessors, while the held-out XOR run was slower than both earlier accessor screens. The results establish a checked compiler-to-proof interface across three accumulator operations but supply no evidence of lower proof-generation time. The agents' journals identify public-continuation composition and general local-frame equality as the next abstraction boundaries.

1 Local frames form a recurring artifact-proof boundary

An exact-artifact proof concerns the instructions decoded from one frozen WebAssembly binary rather than the source program or an asserted compiler result. LeanExe embeds the binary bytes, runs a proved-sound decoder and validator for its accepted profile, translates the validated module to the Talos execution model, and proves a behavioral theorem about that model [1–3, 10]. The compiler and its annotations may generate candidate Lean declarations, but Lean checks every declaration used by the final theorem.

Generated array folds carry their source-level state through WebAssembly parameters, internal locals, and the operand stack. A loop proof needs facts such as the current accumulator, input root, index, effective stop, local-list length, and empty operand stack before it can apply a traversal theorem or reconstruct the invariant. Earlier proofs established these facts with artifact-local theorems closed by reduction, which repeated compiler-layout knowledge inside each behavioral proof.

This work asks whether the compiler's structural knowledge can produce a checked proof interface for those facts. The interface does not state semantic preservation from Lean source to WebAssembly, and it does not add compiler correctness as a premise. It generates exact statements about a frame definition derived from the decoded artifact, then relies on the Lean kernel to check each statement and every later use.

Translation validation and self-certifying compilation also move compiler obligations into checks on a particular output [7, 8]. The present interface has a narrower purpose: it exposes recurring target-level structure to an existing direct behavioral proof. Its value therefore depends on theorem applicability, agent retrieval, proof structure, and proof-generation time rather than on a new end-to-end source theorem.

2 Annotations generate checked accessor families

The LeanExe compiler emits a versioned annotation for each recognized array-fold region. An independent consumer resolves the annotation against the frozen decoded program, checks instruction shape and local roles, and generates exact region equalities and scalar-transition evaluators. For a fold with a recognized guarded back edge, the same consumer now generates the accessor family shown schematically in Figure 1.

The first three generated theorems state the frame's parameter list, internal-local length, and empty operand stack. One further theorem states the result of `Wasm.Locals.get` at each valid combined index, so an artifact

```
<region>_continuing_frame_params
<region>_continuing_frame_locals_length
<region>_continuing_frame_values
<region>_continuing_frame_get_<index> for every combined parameter-and-local index
FixedArrayFold.resultFrame_get_result
FixedArrayFold.resultFrame_get_of_ne
```

Figure 1: The generated continuing-frame declarations and shared result-frame declarations exposed to an artifact proof.

proof can name the compiler role it needs without opening the representation of parameters and internal locals. Lean closes these artifact-specific theorems by reduction against the generated scalar frame.

The result-placement frame belongs to a shared semantic definition rather than one compiler annotation. The theorem `resultFrame_get_result` reads the value written to a valid nonparameter result local, while `resultFrame_get_of_ne` preserves a distinct valid nonparameter getter across the write. Together, the generated and shared theorems cross the continuing-frame and result-placement boundaries without an artifact-local projection lemma.

An application often defines a semantic abbreviation such as `foldFrame` around the generated frame. The reliable use form is `simp only [foldFrame, generated_accessor]`, which unfolds the abbreviation and applies the exact theorem. Unrestricted simplification can reduce `Wasm.Locals.get` into parameter and internal-list branches before the generated theorem matches, recreating the representation proof that the interface was intended to avoid.

3 Structured retrieval connects residual goals to accessors

The annotation recipe records every generated declaration and the shared result-frame theorems. LeanExe also adds a canonical entry to its structured library of lemmas, tactics, and guidance, with overlapping indexes for arrays, compiler motifs, loops, and proof construction. A proof task receives a filtered snapshot of this library and searches its JSON Lines indexes before opening selected entry bodies, an approach evaluated separately for selective discovery [4, 12].

The first addition-fold agent retrieved the accessor entry on its second annotation-directed query. The multiplication-fold agent received the same entry, selected strategy paragraph, and complete recipe, but did not retrieve the entry by name when frame projections appeared later in the proof. It used two generated getters and both shared result-frame theorems while proving eleven other frame facts by direct reduction.

The multiplication journal distinguished a retrieval failure from missing proof support. The proof prompt was therefore changed to treat each new residual goal class as another retrieval checkpoint. Before reducing a frame parameter, local length, operand value, or getter projection, the agent now searches the recipe and relevant library indexes for an exact accessor and records either its use or the reason it does not apply.

The held-out XOR fold tested that instruction. Its journal searched before a capacity-frame projection, found no exact capacity accessor, and recorded the subsequent reduction. At the fold boundary it found and used the generated parameter, local-length, operand-stack, accumulator, root, index, and stop accessors together with both result-frame theorems.

Natural-language proof journals record queries, chosen declarations, failed applications, residual goals, and missing abstractions during construction. The accepted Lean source shows what the proof used, while telemetry records edits and elapsed time. Reading all three artifacts after each run connected the multiplication retrieval failure to the prompt change and connected the XOR proof's remaining work to two general proof boundaries.

4 Three exact-artifact screens establish applicability

The experiments use bounded folds over `Array UInt64`. Inputs of at most eight elements return a singleton containing the left fold, while longer inputs return an empty array. Demo 9 uses wrapping addition with zero, Demo 10 uses wrapping multiplication with one, and the held-out Demo 11 uses bitwise XOR with zero.

Artifact	Operation	Stage 5 (s)	Lines	Words	Retrieval and use
Demo 9	addition	2231	616	2541	Entry retrieved; generated shape and getters used.
Demo 10	multiplication	2099	600	2371	Entry missed; some generated getters and both result getters used.
Demo 11	bitwise XOR	2452	676	2798	Residual-goal search used the generated family on a held-out artifact.

Table 1: Fresh proof-generation screens. Times are rounded monotonic Stage 5 measurements, and word counts are whitespace-delimited source tokens.

Each fresh screen preserved its formal specification, generated source, 1979-byte WebAssembly artifact, decoded Talos program, toolchain, annotation package, and allowed proof library for the duration of that screen. Stage 5 starts when proof generation begins and ends at the first accepted proof, including coding-agent work, its Lean checks, orchestration, and an outer acceptance check. A later invocation of the independent package verifier rebuilt each exact-byte theorem without invoking the coding agent or compiler.

The three proof tasks ran as separate ephemeral sessions under `codex-cli 0.147.0`. The retained packages do not record the served model or reasoning setting inherited by the command, so the elapsed times constitute development observations rather than reproducible model-performance measurements. The fixed-artifact comparisons still identify whether an interface produced an observed improvement in its recorded session, while broader performance claims require repeated runs with complete model and host identities.

The addition proof used generated parameter, local-length, operand-stack, and combined-local getter theorems as well as both shared result-frame theorems. It declared no proof-local frame-projection theorem, and the verifier accepted the exact binary package. Compared with the preceding fold-body reproof over the same artifact, Stage 5 was 7.6 percent slower and the source was 28 lines longer.

The multiplication proof used generated operand-stack and index getters and both result-frame theorems, while its journal exposed the missed retrieval checkpoint. The verifier accepted the exact binary package. Compared with the preceding fold-body reproof, Stage 5 was 9.7 percent slower, the source was 50 lines shorter, and its whitespace-delimited word count increased by 269.

The XOR proof tested the revised retrieval rule on an artifact that had not motivated the accessor interface or prompt edit. The verifier accepted its exact-artifact theorem, and direct execution returned `[15]` for input `[1,2,4,8]` and the empty array for a nine-element input. Its proof stage and source were slower and larger than both preceding accessor screens, although their different accumulator operations and proof histories make those comparisons descriptive.

A separate manual substitution isolates proof structure from agent search. Replacing ten artifact-local projection declarations in the accepted Demo 9 fold-body proof with generated accessors and shared result-frame getters reduced the source from 588 to 555 lines and from 2484 to 2084 whitespace-delimited words. Independent verification accepted the changed proof over the same artifact, but the substitution has no proof-generation-time measurement.

5 The results separate applicability from speed

The accepted addition, multiplication, and XOR proofs establish that one generated accessor schema applies across three scalar operations. The generated theorems remove repeated statements of compiler frame layout, and the manual substitution provides direct evidence of reduced local scaffolding. Promotion of the structured library entry records this recurring applicability rather than a performance claim.

The fresh screens provide no evidence of lower proof-generation time. The fixed-artifact addition and multiplication runs exceeded their immediate predecessors, and the held-out XOR run produced the longest time and largest proof of the three accessor screens. The journals show that proof construction spent substantial effort on length dispatch, allocation, loop initialization, dependent theorem composition, singleton construction, and the public postcondition after the accessor goals had become routine.

The measurements do not support causal comparisons across operations. Coding-agent search varies, each operation has a separate artifact and proof history, and one fresh run per configuration cannot estimate a distribution. Line and word counts describe source structure only imperfectly, while raw bytes penalize descriptive shared declaration names and therefore receive less weight in assessing proof quality.

Two residual boundaries recur in the accepted XOR proof. A compact generated completion adapter could connect the completed loop state to the public singleton-result postcondition without reopening the complete continuation. A general extensionality or equality principle for `Wasm.Locals` could replace remaining reductions that prove equality between semantically identical parameter, local, and operand-stack components.

The next experiment should implement one boundary at a time and screen it first on a fixed artifact, then on a held-out fold. The journal should record whether the agent retrieves the new declaration at the residual goal, whether it avoids local scaffolding, and where the remaining work moves. Retention should consider accepted theorem use, retrieval, revisions, proof structure, and time together, because the accessor experiments show that a useful checked interface can leave total generation time unchanged or worse.

6 Relation to WebAssembly and proof automation

Mechanized WebAssembly semantics and program logics provide foundations for reasoning about target programs [9–11]. LeanExe currently proves closed-module behavior in the pinned Talos semantics and separately connects frozen bytes to the Talos module used by the proof. The frame accessors address proof construction inside that boundary and do not establish equivalence between Talos and the normative WebAssembly semantics.

Verified compilation proves a relation between source and target semantics, while translation validation checks a particular compilation result [5, 8]. Self-certifying WebAssembly compilation can produce checkable witnesses for target properties [7]. LeanExe's generated accessors are smaller checked witnesses about target representation, used indirectly to prove a separate target-level functional theorem.

Lean automation and retrieval systems search established theorem libraries or interact with a prover during construction [6, 12]. This experiment adds two observations for generated artifact proofs: retrieval must recur when the proof exposes a new class of residual goal, and a natural-language journal can identify whether a missed abstraction was absent, undiscovered, or misapplied. The held-out XOR run shows that the revised search instruction worked, while its elapsed time shows that successful retrieval alone does not determine total proof cost.

7 Conclusion

Compiler-generated frame accessors turn exact fold-layout knowledge into Lean declarations checked against a decoded WebAssembly artifact. Shared result-frame theorems extend the interface across accumulator placement, and structured retrieval makes the declarations available when a proof reaches a projection goal. Independent verification accepted addition, multiplication, and held-out XOR proofs that used this interface.

The manual substitution removed ten local declarations and reduced proof source, while the fresh screens supplied no evidence of lower proof-generation time. Journals showed that later composition boundaries dominated the remaining work and exposed one retrieval failure that led to a successful held-out prompt test. The next controlled screens concern a compact loop-completion adapter and a general local-frame equality principle.

References

[1] Cajal Technologies. Talos: A WebAssembly interpreter and verification framework in Lean. Software repository, 2026. URL `https://github.com/cajal-technologies/talos`. LeanExe evaluation revision bb3277e21c9786e3133d5c1601e34ebdc0bea4df.

[2] Andreas Haas, Andreas Rossberg, Derek L. Schuff, Ben L. Titzer, Michael Holman, Dan Gohman, Luke Wagner, Alon Zakai, and J. F. Bastien. Bringing the web up to speed with WebAssembly. In *Proceedings of the 38th ACM SIGPLAN Conference on Programming Language Design and Implementation*, pages 185–200. Association for Computing Machinery, 2017. doi: 10.1145/3062341.3062363.

[3] LeanExe contributors. LeanExe: Compilation and exact-artifact verification for Lean and WebAssembly. Software repository, 2026. URL `https://github.com/jsmorph/leanexe`. Frame-accessor report snapshot 32eeead4c46a097abb5248814ab59d5f92d83d23.

[4] LeanExe contributors. Structured retrieval for a growing Lean artifact-proof library. *marXiv*, 2608.00029, 2026. URL `http://localhost:8000/abs/2608.00029`.

[5] Xavier Leroy. Formal verification of a realistic compiler. *Communications of the ACM*, 52(7):107–115, 2009. doi: 10.1145/1538788.1538814.

[6] Jannis Limperg and Asta Halkjær From. Aesop: White-box best-first proof search for Lean. In *Proceedings of the 12th ACM SIGPLAN International Conference on Certified Programs and Proofs*, pages 253–266. Association for Computing Machinery, 2023. doi: 10.1145/3573105.3575671.

[7] Kedar S. Namjoshi and Anton Xue. A self-certifying compilation framework for WebAssembly. In *Verification, Model Checking, and Abstract Interpretation*, volume 12597 of *Lecture Notes in Computer Science*, pages 127–148. Springer, 2021. doi: 10.1007/978-3-030-67067-2_7.

[8] Amir Pnueli, Michael Siegel, and Eli Singerman. Translation validation. In *Tools and Algorithms for the Construction and Analysis of Systems*, volume 1384 of *Lecture Notes in Computer Science*, pages 151–166. Springer, 1998. doi: 10.1007/BFb0054170.

[9] Xiaojia Rao, Aïna Linn Georges, Maxime Legoupil, Conrad Watt, Jean Pichon-Pharabod, Philippa Gardner, and Lars Birkedal. Iris-Wasm: Robust and modular verification of WebAssembly programs. *Proceedings of the ACM on Programming Languages*, 7(PLDI), 2023. doi: 10.1145/3591265.

[10] Conrad Watt. Mechanising and verifying the WebAssembly specification. In *Proceedings of the 7th ACM SIGPLAN International Conference on Certified Programs and Proofs*, pages 53–65. Association for Computing Machinery, 2018. doi: 10.1145/3167082.

[11] Conrad Watt, Petar Maksimović, Neelakantan R. Krishnaswami, and Philippa Gardner. A program logic for first-order encapsulated WebAssembly. In *33rd European Conference on Object-Oriented Programming*, volume 134 of *Leibniz International Proceedings in Informatics*, pages 9:1–9:30. Schloss Dagstuhl–Leibniz-Zentrum für Informatik, 2019. doi: 10.4230/LIPIcs.ECOOP.2019.9.

[12] Kaiyu Yang, Aidan Swope, Alex Gu, Rahul Chalamala, Peiyang Song, Shixing Yu, Saad Godil, Ryan J. Prenger, and Animashree Anandkumar. LeanDojo: Theorem proving with retrieval-augmented language models. In *Advances in Neural Information Processing Systems 36*, 2023. doi: 10.52202/075280-0944.