Compiler-Generated Frame Accessors in Exact WebAssembly Proofs

Anonymous manuscript

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Abstract

Direct proofs about decoded WebAssembly must establish how compiler-created local frames encode mathematical loop state. LeanExe now emits Lean theorems for the parameter list, local-list length, operand stack, and every index in the parameter-plus-local sequence of each recognized array-fold continuation frame. It places these declarations and two shared theorems for reading locals after result placement in a structured library retrieved by a coding agent. A manual substitution within a proof of the same binary removed ten proof-local projection declarations and reduced proof source from 588 to 555 lines. Three separate ephemeral coding-agent sessions then proved the behavior of frozen WebAssembly binaries for bounded addition, multiplication, and bitwise-XOR folds, and a separate verifier accepted all three. The direct artifact-proof stages took 2231, 2099, and 2452 seconds. The addition and multiplication runs were slower than their same-binary predecessors, while the held-out XOR run was slower than both earlier runs using the accessors. The results establish a checked compiler-to-proof interface across three accumulator operations but supply no evidence of lower proof-generation time. The agents' journals identify proof obligations that connect loop completion to the public postcondition and prove equality between WebAssembly local frames.

1 Local frames form a recurring artifact-proof boundary

An exact-artifact proof concerns the instructions decoded from one frozen WebAssembly binary rather than the source program or an asserted compiler result. LeanExe embeds the binary bytes, runs a proved-sound decoder and validator for its accepted profile, translates the validated module to the Talos execution model, and proves a behavioral theorem about that model [1–3, 10]. The compiler and its annotations may generate candidate Lean declarations, but Lean checks every declaration used by the final theorem.

Generated array folds carry their source-level state through WebAssembly parameters, internal locals, and the operand stack. A loop proof needs facts such as the current accumulator, input root, index, effective stop, local-list length, and empty operand stack before it can apply a traversal theorem or reconstruct the invariant. Earlier proofs established these facts with artifact-local theorems closed by reduction, which repeated compiler-layout knowledge inside each behavioral proof.

This work asks whether the compiler's structural knowledge can produce a checked proof interface for those facts. The interface does not state semantic preservation from Lean source to WebAssembly, and it does not add compiler correctness as a premise. It generates exact statements about a frame definition derived from the decoded artifact, then relies on the Lean kernel to check each statement and every later use.

Translation validation and self-certifying compilation also move compiler obligations into checks on a particular output [7, 8]. The present interface has a narrower purpose: it exposes recurring target-level structure to an existing direct behavioral proof. Its value therefore depends on theorem applicability, agent retrieval, proof structure, and proof-generation time rather than on a new end-to-end source theorem.

2 Annotations generate checked accessor families

The LeanExe compiler emits a versioned annotation for each recognized array-fold region. An independent consumer resolves the annotation against the frozen decoded program, checks instruction shape and local roles, and generates exact region equalities and Lean theorems that evaluate each scalar operation and control transition. For a fold with a recognized guarded back edge, the same consumer now generates the accessor family shown schematically in Figure 1.

```
<region>_continuing_frame_params
<region>_continuing_frame_locals_length
<region>_continuing_frame_values
<region>_continuing_frame_get_<index> for every index in parameters followed by locals
FixedArrayFold.resultFrame_get_result
FixedArrayFold.resultFrame_get_of_ne
```

Figure 1: The generated continuing-frame declarations and shared result-frame declarations exposed to an artifact proof.

The first three generated theorems state the frame's parameter list, internal-local length, and empty operand stack. A `Wasm.Locals` index enumerates parameters first and internal locals second, and one further theorem states the result of `Wasm.Locals.get` at each valid index in that sequence. Lean closes these artifact-specific theorems by reduction against the generated scalar frame.

The result-placement frame belongs to a shared semantic definition rather than one compiler annotation. The theorem `resultFrame_get_result` reads the value written to a valid nonparameter result local, while `resultFrame_get_of_ne` preserves a distinct valid nonparameter getter across the write. Together, the generated and shared theorems cross the continuing-frame and result-placement boundaries without an artifact-local projection lemma.

An application often defines a semantic abbreviation such as `foldFrame` around the generated frame. The reliable use form is `simp only [foldFrame, generated_accessor]`, which unfolds the abbreviation and applies the exact theorem. Unrestricted simplification can reduce `Wasm.Locals.get` into parameter and internal-list branches before the generated theorem matches, recreating the representation proof that the interface was intended to avoid.

3 Structured retrieval connects residual goals to accessors

The generated `PROOF_RECIPES.json` file lists every applicable declaration and explains the purpose of each one. LeanExe also adds a canonical entry to its structured library of lemmas, tactics, and guidance, with overlapping indexes for arrays, compiler motifs, loops, and proof construction. The orchestrator gives a coding agent a filtered library snapshot, and the agent searches its JSON Lines indexes before opening selected entry bodies. The earlier study of structured lemmas, tactics, and guidance (LTG) evaluates this selective-discovery procedure [4, 12].

The first addition-fold agent retrieved the accessor entry on its second annotation-directed query. The multiplication-fold agent received the same entry and selected strategy paragraph, while `PROOF_RECIPES.json` listed every exact accessor, but the agent did not retrieve the entry by name when frame projections appeared later in the proof. It used two generated getters and both shared result-frame theorems while proving eleven other frame facts by direct reduction.

Our review of the multiplication journal distinguished a retrieval failure from missing proof support. We therefore changed the proof prompt to treat each new residual goal class as another retrieval checkpoint. Before reducing a frame parameter, local length, operand value, or getter projection, the agent now searches `PROOF_RECIPES.json` and the relevant library indexes for an exact accessor and records either its use or the reason it does not apply.

We tested the revised instruction in the held-out XOR run. The capacity frame records the WebAssembly locals after a length branch computes the allocation capacity, while the generated accessor family covers the later array-fold continuation frame. The agent recorded that no exact capacity accessor existed before reducing that projection, then used the generated parameter, local-length, operand-stack, accumulator, root, index, and stop accessors at the fold boundary together with both result-frame theorems.

Natural-language proof journals record queries, chosen declarations, failed applications, residual goals, and missing abstractions during construction. The accepted Lean source shows what the proof used, while telemetry records edits and elapsed time. Reading the journal, accepted Lean source, and telemetry after each run connected the multiplication retrieval failure to the prompt change and connected the XOR proof's remaining obligations to two general proof boundaries.

Artifact	Operation	Stage 5 (s)	Lines	Words	Retrieval and use
Demo 9	addition	2231	616	2541	Agent retrieved the entry and used generated shape and getter theorems.
Demo 10	multiplication	2099	600	2371	Agent missed the entry but used some generated getters and both result getters.
Demo 11	bitwise XOR	2452	676	2798	Agent repeated retrieval and used the generated family on a held-out artifact.

Table 1: Fresh proof-generation runs. Times are rounded monotonic Stage 5 measurements, and word counts are whitespace-delimited source tokens.

Leanexegen divides generation into eight reported stages, and Stage 5 is the direct artifact-proof stage. Its elapsed measurement starts when proof generation begins and ends at the first accepted proof, including coding-agent work, the agent's Lean checks, orchestration, and an outer acceptance check. Table 1 reports this monotonic Stage 5 measurement.

4 Three exact-artifact runs establish applicability

The experiments use bounded folds over `Array UInt64`. Each program returns a singleton containing the left fold for an input of at most eight elements and returns an empty array for a longer input. Demo 9 uses wrapping addition with zero, Demo 10 uses wrapping multiplication with one, and the held-out Demo 11 uses bitwise XOR with zero.

Each fresh run preserved its formal specification, generated source, WebAssembly binary, decoded Talos program, toolchain, annotation package, and allowed proof library for the duration of that run. A later invocation of the independent package verifier rebuilt all three exact-byte theorems without invoking the coding agent or compiler. The retained verification packages contain the binary bytes, generated declarations, accepted proof, library snapshot, journal, telemetry, and files that identify the artifact and tool versions used for this check.

The three proof tasks ran as separate ephemeral sessions under `codex-cli 0.147.0`. The retained packages do not record the served model or reasoning setting inherited by the command, so the elapsed times constitute development observations rather than reproducible model-performance measurements. The same-binary comparisons record whether the session using the interface took more or less time than its predecessor, while broader performance claims require repeated runs with complete model and host identities.

The same-binary baselines come from the retained Demo 9 and Demo 10 packages named `experiments/fold-body-reproof.proof` under their respective demo directories in the cited software snapshot [3]. Each package's `proof-telemetry.json` records the predecessor time, and `proof-timings.json` records the source measurements used in the comparison. Demo 9's predecessor took 2074 seconds and contained 588 lines, while Demo 10's predecessor took 1913 seconds and contained 650 lines.

The addition proof used generated parameter, local-length, and operand-stack theorems together with generated getter theorems. It also used both shared result-frame theorems and declared no proof-local frame-projection theorem. Compared with the preceding fold-body reproof over the same artifact, Stage 5 was 7.6 percent slower and the source was 28 lines longer.

The multiplication proof used generated operand-stack and index getters and both result-frame theorems. Its journal also exposed the missed retrieval checkpoint described in Section 3. Compared with the preceding fold-body reproof, Stage 5 was 9.7 percent slower, the source was 50 lines shorter, and its whitespace-delimited word count increased by 269.

We tested the revised retrieval rule in the XOR run on an artifact that had not motivated the accessor interface or prompt edit. The agent used each generated frame-shape class needed at the fold boundaries and used both shared result-frame theorems. The run took more time and produced more proof lines than both preceding accessor runs, although their different accumulator operations and proof histories make those comparisons descriptive.

A separate manual substitution isolates proof structure from agent search. Replacing ten artifact-local projection declarations in the accepted Demo 9 fold-body proof with generated accessors and shared result-frame getters reduced the source from 588 to 555 lines and from 2484 to 2084 whitespace-delimited words. Independent verification accepted the changed proof over the same artifact, but the substitution has no proof-generation-time measurement.

5 The results separate applicability from speed

The accepted addition, multiplication, and XOR proofs establish that one generated accessor schema applies across three scalar operations. The generated theorems remove repeated statements of compiler frame layout, and the manual substitution provides direct evidence that they remove proof-local declarations. The catalog now marks the accessor entry as promoted, which makes retrieval favor it because accepted proofs use it across three operations. This status records recurring applicability rather than a performance claim.

The fresh runs provide no evidence of lower proof-generation time. The same-binary addition and multiplication runs exceeded their immediate predecessors, and the held-out XOR run produced the longest time and largest proof of the three accessor runs. After the agents resolved the accessor goals, their journals record further proof work on length dispatch, allocation, loop initialization, dependent theorem composition, singleton construction, and the public postcondition.

The measurements do not support causal comparisons across operations. Coding-agent search varies. Each operation has a separate artifact and proof history, and one fresh run per configuration cannot estimate a distribution. Line and word counts describe source structure only imperfectly.

Two residual boundaries recur in the accepted XOR proof. A compact generated theorem could connect the completed loop state to the public singleton-result postcondition without reopening all instructions and proof obligations after the loop. A general extensionality or equality principle for `Wasm.Locals` could replace remaining reductions that prove equality between semantically identical parameter, local, and operand-stack components.

The next experiment should implement one boundary at a time and test it first on a fixed artifact, then on a held-out fold. The journal should record whether the agent retrieves the new declaration at the residual goal, whether it avoids new proof-local declarations, and which proof obligations remain. A decision to retain or promote the new theorem and guidance should consider accepted theorem use, retrieval, revisions, proof structure, and time together, because the accessor experiments show that a useful checked interface can produce unchanged or longer total generation time.

6 Relation to WebAssembly and proof automation

Watt mechanizes the WebAssembly specification, while Wasm Logic and Iris-Wasm prove sound program logics for classes of WebAssembly programs [9–11]. LeanExe instead proves closed-module behavior in the pinned Talos semantics and separately connects frozen bytes to the Talos module used by the proof. The frame accessors support construction of the Talos behavioral proof and do not establish equivalence between Talos and the normative WebAssembly semantics.

Verified compilation proves a relation between source and target semantics, while translation validation checks a particular compilation result [5, 8]. Self-certifying WebAssembly compilation can produce checkable witnesses for target properties [7]. LeanExe's generated accessors are smaller checked witnesses about target representation, used indirectly to prove a separate target-level functional theorem.

Lean automation and retrieval systems search established theorem libraries or interact with a prover during construction [6, 12]. In the reported proof procedure, the agent now repeats retrieval when Lean exposes a new residual-goal class, and the natural-language journal records whether a missed declaration was absent, undiscovered, or misapplied. The held-out XOR run supplies one successful test of that revised instruction, while its elapsed time shows that successful retrieval alone does not determine total proof cost.

7 Conclusion

Compiler-generated frame accessors turn exact fold-layout knowledge into Lean declarations checked against a decoded WebAssembly artifact. Shared result-frame theorems extend the interface across accumulator placement, and structured retrieval makes the declarations available when a proof reaches a projection goal. Independent verification accepted addition, multiplication, and held-out XOR proofs that used this interface.

The manual substitution removed ten local declarations and reduced proof source, while the fresh runs supplied no evidence of lower proof-generation time. Journals recorded later composition obligations and exposed one retrieval failure that led to a successful held-out prompt test. The next controlled runs concern a compact theorem from loop completion to the public postcondition and a general equality principle for WebAssembly local frames.

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