

Complete secp256k1 Point Addition with 1,601 Logical Qubits

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Abstract

VQ provides a Lean-verified complete secp256k1 point-adder using 1,601 logical qubits and at most 3,070,200 Toffoli-class gates. The program adds a runtime classical point to a quantum accumulator, restores all 1,089 workspace qubits to zero, and handles the group identity, doubling, and inverse points. Its measured execution preserves finite superpositions with a common branch amplitude, and the correct output has probability one on every valid basis input. Checked resource theorems bound the Toffoli-class count on every execution path and its expectation by 3,070,200, with at most 1,959,699 measurements. The program uses 13,342 mutable classical bits and 512 immutable classical input bits.

1 Complete point addition

VQ’s complete runtime point-adder implements secp256k1 group translation with 1,601 logical qubits and a checked upper bound of 3,070,200 Toffoli-class gates. The construction and its proofs belong to the algorithm library described in the VQ system report [1].

Let E denote the secp256k1 group on $y^2 = x^3 + 7$ over $\mathbb{F}_p$, where $p = 2^{256} - 2^{32} - 977$, and let O denote its identity. The encoding enc stores canonical affine coordinates in two 256-bit registers and represents O by $(0, 0)$. For every $Q \in E$ supplied as a fixed classical input, the quantum action on valid encodings is

$$|\text{enc}(P)\rangle|0^{1089}\rangle \mapsto |\text{enc}(P + Q)\rangle|0^{1089}\rangle, \quad P \in E. \quad (1)$$

The same program accepts every valid Q at runtime and preserves its classical input. Its mutable classical register starts at zero. The group-law theorem identifies the encoded operation with addition in Mathlib’s affine point group. Completeness includes $P = O$, $Q = O$, $P = Q$, and $P = -Q$.

Allocation	Bits or qubits
Logical qubits	1,601
Accumulator qubits	512
Workspace qubits	1,089
Mutable classical bits	13,342
Immutable classical input bits	512

Table 1: Register allocation. All quantum workspace starts and ends at zero.

The resource model counts a controlled-controlled- Z gate as one Toffoli-class gate. Hadamard conjugation converts that gate to a Toffoli gate. Classical preparation and feed-forward execute in the program’s classical instruction semantics. The reported quantities describe the logical program before error-correction or hardware scheduling.

2 Arithmetic and exceptional cases

The program prepares classical coordinates and correction data from Q , then branches on whether $Q = O$. The identity branch leaves the quantum state unchanged. For a nonidentity addend $Q = (a, b)$, the quantum computation tags exceptional accumulator values, applies an affine arithmetic sequence, corrects the tagged outputs, and erases the tags. The runtime program includes this preparation and branch.

For an accumulator (x, y) , the arithmetic sequence subtracts the addend's coordinates, divides, subtracts a square, adds $3a$, multiplies, and finishes the coordinate transformation. Its value formula is

$$\begin{aligned} d &= x - a, & \lambda &= (y - b) \sigma(d)^{-1}, & u &= d - \lambda^2 + 3a, \\ (x', y') &= (a - u, \sigma(u)\lambda - b), \end{aligned} \tag{2}$$

with arithmetic in $\mathbb{F}_p$ and $\sigma(t) = 1$ when $t = 0$, $\sigma(t) = t$ otherwise. Replacing a zero factor by one gives an invertible multiplication on the other field register. The measured arithmetic component uses recorded Euclidean computation and replay. The squaring component uses a measured Karatsuba construction.

For $Q \neq O$, the four exceptional input cases are $P = O$, $P = Q$, $P = -Q$, and $P = -2Q$. Their required outputs are Q , $2Q$, O , and $-Q$, respectively. The fourth case corrects the replacement of a zero multiplier by one in the final multiplication. Classical preparation computes the coordinates of these four input and output pairs and the bitwise correction masks. Quantum comparisons record the case tags before arithmetic. After correction, comparisons against the translated outputs erase those tags. The composition proof establishes the resulting complete group action with the workspace restored.

Component	Toffoli-class gates	Measurements
Division and multiplication	2,953,930	1,893,066
Karatsuba square subtraction	66,642	66,633
Coordinate additions, subtractions, and reflection	16,962	0
Exceptional-case tagging, correction, and erasure	32,666	0
Total for the nonidentity branch	3,070,200	1,959,699

Table 2: Exact static counts for the prepared nonidentity branch.

Each multiplication or division contributes 1,476,965 Toffoli-class gates and 946,533 measurements. The arithmetic sequence has 3,037,534 Toffoli-class gates. The complete prepared program adds the exceptional-case operations to obtain Table 2. Runtime classical preparation contributes zero to these two quantum counts. Branching on the identity addend gives the complete program the static count intervals $[0, 3,070,200]$ and $[0, 1,959,699]$.

3 Coherence, cleanup, and probability

The measured execution satisfies a common-amplitude theorem on finite superpositions of valid accumulator states. VQ represents amplitudes exactly in

$$D_\ell = (\mathbb{Z}[X]/(X^{2^{\ell-1}} + 1))[1/2], \quad \ell \geq 3,$$

with the distinguished complex interpretation $X = e^{2\pi i/2^\ell}$. For a fixed valid classical addend Q and an initial quantum state

$$|\psi\rangle = \sum_{j=1}^r \alpha_j |\text{enc}(P_j)\rangle |0^{1089}\rangle, \quad \alpha_j \in D_\ell, \quad P_j \in E,$$

the **superposition** theorem gives every terminal measurement branch the unnormalized quantum state

$$2^{-M(Q)/2} \sum_{j=1}^r \alpha_j |\text{enc}(P_j + Q)\rangle |0^{1089}\rangle, \quad M(Q) = \begin{cases} 0 & Q = O, \\ 1,959,699 & Q \neq O. \end{cases} \quad (3)$$

The branch factor depends only on the classical addend and preserves relative amplitudes, including phases. Every branch preserves Q and restores all quantum workspace. The theorem permits an arbitrary initial outcome record and requires a zero-initialized mutable classical register.

For each valid basis input, **success_probability** proves that the terminal event identifying the translated basis state has probability one. Its proof combines the branch-state equation with the program’s well-formedness and the normalization theorem for measured execution. The **complete** theorem packages this probability result with quantum cleanup, the allocation in Table 1, and logical-resource bounds.

The resource proofs connect static counting to execution paths and their probabilities. The theorem **path_toffoli_upper** bounds every path. The theorem **expected_toffoli_upper** proves $\mathbb{E}[T] \leq 3,070,200$ for the Toffoli-class count T . The expectation is over measurement outcomes for a fixed basis input and classical addend. Multiplication by the fixed allocation gives $1,601 \mathbb{E}[T] \leq 4,915,390,200$. Both statements concern the same runtime program as the semantic theorems.

4 Formal sources and verification

The result’s aggregate module is `VQ.Curve.RecordedAffine.WideRuntimeComplete` at repository revision `7cfc9dee` [2]. The retained successful Lean build checks this aggregate and its guarded axiom declarations with Lean 4.34.0-rc2. The guarded declarations for **complete**, **superposition**, and **success_probability** report only `propext`, `Classical.choice`, and `Quot.sound`. The verification record identifies the checked source revision as `7a494241`. The formal library, toolchain, and Lake configuration at the revision cited here are byte-identical to that checked revision. The VQ system report [1] describes the exact semantics, resource accounting, and library organization used by these proofs.

References

- [1] Jamie Stephens and AI. *VQ: Quantum Algorithm Development and Verification with an Evolving Knowledge Base*. [arXiv:2608.00049v17](https://arxiv.org/abs/2608.00049), 21 September 2026.
- [2] VQ repository. Revision `7cfc9dee88db6431b25b36401aa60d0aadcb82a4`. Source links in the text resolve at this revision.