

Checked WGS� Compilation and Conditional GPT-2 Execution in LeanExe

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Abstract

The goal of LeanExe is to run an algorithm written in Lean and obtain a checked proof that the emitted program performs that algorithm. Previous work established this connection for cached GPT-2 inference in a formal WebAssembly execution model. The WGS� work extends the approach toward GPU execution by moving matrix products to the WebGPU Shading Language (WGS�) while retaining the model controller in WebAssembly. We present a compiler for a restricted class of Lean definitions that checks each translation against an independent shader parser and a statement-execution semantics. Packed-layout proofs connect six compiled shaders to the matrix operations of a 124M-parameter GPT-2 implementation. The hybrid session theorem equates cache and logit bytes with the Lean cached-step recurrence for arbitrary weight bytes of the required length and up to 128 valid tokens, assuming completed shader execution with separate FP32 arithmetic and exact host transfers. The branch supplies a native inference runner and records a CPU WebGPU comparison in which all 6,432,896 logits match the parent WebAssembly implementation bit for bit. Runtime conformance, hybrid binary decoding, and browser execution remain separate verification or evaluation tasks.

1 Goal, motivation, and outcome

1.1 Lean programs and GPU execution

LeanExe’s overall goal is to execute an algorithm specified in Lean and obtain a checked proof that the emitted program computes the same result. For neural-network inference, that result includes the floating-point output words and updated model state. The specification fixes operation order, tensor representation, and the rules for updating the cache. The preceding GPT-2 development applied this approach to a pretrained 124M-parameter model with a 128-token context, using WebAssembly as the execution target [1].

The WGS� work was undertaken to extend this verification approach to GPU execution. A matrix product’s output coordinates can be computed independently, making those operations a target for parallel execution. The development plan selected FP32 matrix multiplication for the first compute kernels and retained control flow and the remaining model operations in WebAssembly [2]. WebGPU provides the compute interface, and WGS� specifies the shader programs submitted through it. The intended benefit is faster matrix computation through GPU parallelism while preserving a proof connection to the Lean algorithm. The execution evidence in this report comes from a CPU implementation of WebGPU.

Moving a matrix operation to a shader creates proof obligations at both the computation and calling interface. The emitted shader must implement the selected Lean operation with the required indexing and arithmetic order. The combined program must supply the right bytes, complete the

dispatch, return the result to Wasm memory, and preserve the state needed by the next operation. WGSL’s permitted floating-point choices also require an explicit arithmetic assumption for exact word equality [5]. The work therefore develops shader certificates, packed-layout proofs, and controller proofs that compose through a stated host interface.

1.2 Hybrid GPT-2 implementation and conditional session theorem

The branch supplies a compiler for a bounded subset of Lean, a native GPT-2 inference runner combining Wasm and WGSL, and generated proofs relating that combined program to the parent Lean specification. Its six shader definitions cover the four matrix-product roles in each transformer block and the two slices of the vocabulary projection. The integration journal records successful proof checks, text completions, and a full 128-context CPU comparison with the parent Wasm implementation [15, 16]. The proofs connect shader computation and the Wasm controller to the Lean algorithm under the execution assumptions stated below. Physical-GPU behavior and browser execution remain evaluation tasks.

The hybrid session theorem equates cache and logit bytes with the Lean cached-step recurrence for arbitrary weight bytes of the required length and up to 128 valid tokens, assuming completed shader execution with separate FP32 arithmetic and exact host transfers. The public declarations are `Project.Gpt2Hybrid.Spec.cachedStep_exact`, `gpt2_128_exact`, and `gpt2_128_exact_for`. A build generates these declarations by replaying the parent controller proofs against the hybrid module and the six shader certificates. The integration journal records their successful Lean checks and axiom audits [11, 16].

The parent report establishes an exact-binary WebAssembly result for the same Lean algorithm [1]. Its specification is `LeanExe.Models.Gpt2.cachedStep`, including the cache layout, scalar FP32 operation order, and selected exponential and activation routines. Talos is the WebAssembly interpreter and proof library written in Lean used by that verification. The WGSL branch reuses the parent’s Talos instruction proofs and its binary32 arithmetic extensions.

The reviewed source is the `wgsl` branch at revision `ba9f02ce930243a3d35d4d10df6f9c97850ca1e8`. It pins Lean 4.34.0-rc2 and Talos revision `87e3aa5e8f6e6f3b3eb5e7e4c5aba43071002d47`. The earlier report’s version 7 also proves decoding and validation of a frozen parent WebAssembly binary. The hybrid changes that binary by adding imports and replacing matrix implementations. Its present proof starts from the module declarations produced by an external parser. Extending the parent’s binary certificate to those changed bytes remains a separate obligation [1, 13].

The runtime keeps embedding, normalization, attention, activations, residuals, cache construction, and control flow in Lean-generated WebAssembly. Four products per transformer block and two vocabulary slices execute as WGSL. The model has twelve blocks, hidden width 768, feed-forward width 3072, and 50,257 vocabulary entries. Weights occupy 497,759,232 bytes. Each token extends the cache by 73,728 bytes and returns 201,028 logit bytes [15]. These sizes determine the theorem’s memory and index premises.

2 Per-compilation shader certificates

2.1 Lean source subset

The compiler accepts a monomorphic annotated Lean definition of type

$$\text{ScalarArithmetic} \rightarrow (\mathbb{N} \rightarrow \text{UInt32}) \rightarrow (\mathbb{N} \rightarrow \text{UInt32}) \rightarrow \mathbb{N} \rightarrow \mathbb{N} \rightarrow \text{UInt32}.$$

The five arguments supply scalar operations, input buffers A and B , and output coordinates. A `UInt32` represents a floating-point word. The `add` and `mul` fields of `ScalarArithmetic` specify its arithmetic [3]. For example, an ordered matrix cell is expressed by

```
@[wgs1] def matrix : Source.Kernel := fun ar a b row col =>
  Source.fold 4 0 fun k acc =>
    ar.add acc (ar.mul (a (row * 4 + k)) (b (k * 3 + col)))
```

The definition’s elaborated body determines the emitted computation. Transparent helpers, word and index bindings, nested literal-count folds, finite word constants, buffer reads, addition, and multiplication belong to the supported subset. Index expressions use naturals, coordinates, fold indices, addition, and multiplication. The implementation bounds normalization and rejects unsupported residual expressions. Runtime branches, dynamic iteration, barriers, atomics, and shared-memory communication lie outside this grammar [3].

Every fold count is at most 65,535. Static access checks bound each index expression and its intermediate arithmetic, preventing u32 overflow and out-of-range reads. Constants must encode finite binary32 values, including constants in unused bindings. Input buffer words remain unrestricted. The emitted wrapper declares two read-only storage buffers, a distinct output buffer, an 8×8 workgroup, an edge guard, and one output store per active invocation [3, 4].

2.2 Parsing, source equality, and execution

The checker independently parses the emitted shader string into a statement program. It checks the declarations, bindings, guard, loop initializer, comparison, increment, accumulator assignment, lexical scope, final store, and complete token consumption. Parsed locals become indexed slots. Bindings and loops remain statements in the syntax tree. The parser accepts the compiler’s restricted WGS� grammar [3].

For source function f , shader text s , parsed statement program P , and shape S , the compiler constructs three certificates:

$$\text{parse}(s) = \text{ok}(P), \quad (1)$$

$$f = \text{interpretSource}(P), \quad (2)$$

$$\text{Implements}(s, S, f). \quad (3)$$

Equation (2) quantifies over both input buffers, both coordinates, and the scalar interpretation. Its proof checks equality to the named Lean definition. An incorrect arithmetic operator or a custom natural-number addition instance therefore has to satisfy the original source equality before the checker can accept the shader [3].

The `Implements` proposition quantifies over scalar interpretation a , memory M , output size c , and u32 coordinates (r, q) . With adequate buffer lengths, it states

$$\text{runShader}(s, a, M, c, r, q) = \text{ok}(\text{expected}(S, f, a, M, r, q)).$$

For an active coordinate, `expected` is a write of $f(a, M.A, M.B, r, q)$ to $rS.\text{cols} + q$. For an inactive coordinate, it is no write. The execution function can fail on a missing local, invalid load or store, or exhausted loop budget. The theorem rules out each modeled failure [4].

The shared proof establishes agreement between wrapping u32 index calculations and natural-number source indices within the validated bounds. It then proves statement execution equal to source interpretation, including local bindings and scope restoration. A loop tests its u32 counter,

executes its body, assigns the accumulator, and increments the counter. Its budget equals the literal count, and the proof establishes that the guard becomes false before an additional body execution is needed. This is a successful-execution result with bounded termination [4].

Checking finishes synchronously before files are emitted. The parser reductions use kernel evaluation. The compiler audits transitive dependencies of all three public certificates and permits only `propext`, `Classical.choice`, and `Quot.sound`. It saves the shader, metadata, and Lean declarations that can be checked again independently of compilation. The command `#check_wgsl` applies the same proof requirements to an existing shader file [3]. File reading and writing remain external operations around the theorem about the embedded string.

2.3 Dispatch and arithmetic scope

The body-compiler proof establishes one invocation’s semantics. Separate geometric lemmas prove coverage by the rounded-up dispatch and distinct output addresses for distinct active cells. The operational model assumes read-only inputs and a separate output buffer. A complete refinement to the WebGPU execution model, including an external scheduler, remains outside these certificates [4].

The strict GPT-2 instantiation uses the pure Talos binary32 addition and multiplication functions, with source-ordered accumulation and separate rounding. Its specification includes signed zeros, subnormals, and exceptional words. WGSL permits implementation choices involving reassociation, fusion, subnormals, zero signs, and exceptional values [5]. Applying the exact theorem to a runtime therefore requires more than acceptance of the shader by that runtime: execution must implement the stated operations and preserve the represented words through loads, copies, and stores.

The branch also contains an earlier fixed-template GEMM development. That path proves termination and output correspondence under arbitrary interleavings in its own dispatch model, and supports separate or locally fused accumulation. A fusion theorem characterizes each output by a sequence of explicit per-step choices. Its numerical bounds and small-model mixed-precision experiments use additional domain premises [6]. The current body compiler has its own source and statement proofs. The older scheduler, fusion, and numerical theorems do not supply those properties for the new grammar.

3 Connection to packed GPT-2 operations

3.1 Ordered products and byte layouts

The current integration compiles six kernel definitions. The four block products append bias words to their matrix input view and perform bias addition after the dot product. Table 1 gives the shapes, all with one output row [7].

Operation	Inner dimension	Output columns	Final bias
QKV projection	768	2304	Yes
Attention projection	768	768	Yes
Feed-forward expansion	768	3072	Yes
Feed-forward projection	3072	768	Yes
Vocabulary, first slice	768	25129	No
Vocabulary, second slice	768	25128	No

Table 1: Shader roles in each cached token step. The four block roles repeat across twelve blocks.

Let $\text{word}(W, i)$ read word i from packed byte array W . For matrix offset o , bias offset b , inner dimension K , and column count N , the shader’s second input view is

$$B(i) = \begin{cases} \text{word}(W, o + i), & i < KN, \\ \text{word}(W, b + i - KN), & i \geq KN. \end{cases}$$

The source proof identifies the reads at $kN + q$ with the parent’s matrix reads, and the read at $KN + q$ with its bias. Induction over the fold proves equality to the parent’s ordered dot-product prefix. A separate arithmetic equality connects the Lean source’s logical FP32 operations to the pure Talos operations for all input words [7].

The tied token embedding is vocabulary-major in the parent weights. A vocabulary shader uses the transposed view

$$B_s(kN + q) = \text{word}(W, (s + q)768 + k),$$

with $(s, N) = (0, 25129)$ or $(25129, 25128)$. The split assigns each vocabulary column to one complete 768-term dot product. The composition theorem selects its shader, local column, and shifted output address, preserving the operation order within every dot product [7].

The resulting theorems `biasedBytes_exact` and `vocabularyBytes_exact` identify serialized shader results with the parent’s `linearRows` and `vocabularyHead` byte arrays. The serializer’s total definition maps an unsuccessful shader result to a default word. The preceding per-column theorem proves success at every serialized column, so the byte equality eliminates that default case [8]. These are equalities of word and byte computations without weight-magnitude premises.

3.2 Host imports and controller reuse

The hybrid module adds imports `wgsl.linear` and `wgsl.vocabulary`. Its Wasm wrappers allocate packed output storage and call the imports. Function indices shift by two. The proof generator checks the wrappers and unchanged function declarations against the independently parsed hybrid module [13, 9].

The reusable `ImportShift` relation identifies a closed set of unchanged functions, their renamed direct calls, and the memory declaration. Its semantic transport theorem preserves interpreter execution at every fuel value and therefore transports the parent’s total-correctness statements. Functions in the selected set cannot call the changed matrix implementations. Controller segments outside that set are checked again, with the replacement matrix theorems at their call sites [10, 11].

The interface `PackedBackend.Kernels` contains the six shader texts and their `Implements` proofs. `PackedBackend.Code` identifies the wrappers, allocator, memory mode, and imports in the module. `PackedBackend.Host` specifies what the external calls do. For represented and protected input buffers with adequate sizes and allocation resources, it requires the host to return the modeled shader-output bytes in the allocated output and preserve the remaining store [9].

The host premise concerns execution of the certified shader semantics. Equality of those shader bytes to the parent matrix result follows from the packed-layout and arithmetic theorems. Combining the two establishes exact returned lengths and bytes, ownership of the output, preservation of protected memory, heap validity, and memory-capacity preservation. Those postconditions supply the parent’s controller proofs with the facts needed for subsequent calls and releases [8, 9].

4 Conditional session theorem and deployment boundary

Let W be any 497,759,232-byte array and let $t_0, \dots, t_{n-1}$ satisfy $t_i < 50257$ and $n \leq 128$. Define the source trace by

$$C_0 = \epsilon, \quad (C_{i+1}, L_i) = \text{cachedStep}(W, C_i, t_i, i).$$

Here ϵ is the empty cache, and L_i is the complete logit byte array. Under `PackedBackend.Host`, the generated `gpt2_128_exact` theorem composes reset, weight allocation, the mathematical write of W , successive inference calls, logit availability, and releases with this source trace. Its session predicate observes cache and logit bytes in memory. Its call predicate, `TerminatesWith`, requires success for every sufficiently large interpreter fuel [11, 12].

The session derives its heap and buffer preconditions from initialization and preserves them across token calls. The cached-step proof also covers rejected input lengths, token IDs, and positions. The algorithm uses arbitrary weight words, including exceptional encodings. Consequently, the host premise must hold for the corresponding strict word semantics if the theorem is applied on that full domain. The source theorem supplies no assertion that a particular GPU satisfies this premise on all weight arrays.

Wasmtime is the standalone WebAssembly runtime used by the native host. The host enables its Cranelift arithmetic NaN canonicalization mode, as in the parent implementation. Native WebGPU dispatch and readback finish before an import returns to Wasm. GPU weight views use the matrix offsets passed by Wasm, and the host invalidates cached views after overlapping external writes or reset. A model session uses fifty matrix views [14].

The browser implementation runs Wasm in a worker. Each import copies input words to shared staging, sends a dispatch request, and waits for completion notification. The WebGPU host writes result bytes to staging before waking the worker. Each browser run creates its own host and copies the same weight bytes into Wasm and the host’s source buffer. The source and syntax checks exist, but the branch records no completed browser execution test [14, 16].

Tokenization, sampling, and the FP32-to-FP64 adapter that presents logit words to the Wasm sampler lie outside the inference theorem [13].

The hybrid theorem’s axiom audit permits the same three standard logical axioms as the parent. Its explicit `Host` hypothesis is additional to that logical axiom list. An axiom audit alone cannot discharge the hypothesis [11].

5 Execution evidence and review findings

5.1 Recorded native tests

The integration journal records execution of the pretrained checkpoint on SwiftShader’s CPU WebGPU device. Across a forced 128-token sequence, all 6,432,896 candidate logit words matched the parent Wasm outputs. Complete cache arrays matched at positions 0, 1, 126, and 127. Invalid-input tests preserved allocator statistics and memory size, and cache restart tests reproduced the outputs. The test implementation checks word equality explicitly [16, 17].

The recorded PyTorch comparison reported agreement of all 128 argmax choices and a maximum absolute logit difference of approximately 0.001541. Every logit satisfied the test tolerance $0.002 + 0.0001|\text{reference}|$. The combined parent, hybrid, and reference comparison took approximately 114 seconds. This timing describes that test workload. It supplies no physical-GPU throughput estimate or general numerical error bound [16, 17].

The journal also records greedy text completions and a sampled parent/hybrid pair using the same Wasm tokenizer and sampler, temperature 0.8, and seed 42. Both the sampled text and

logged logit bytes matched. The rebuilt source bundle reproduced these traces and passed focused tokenizer, sampler, and invalid-request tests. Tokenization and sampling remain outside the formal inference claim [16].

The branch records eleven body-compiler examples with three certificates each and 63 exact CPU WebGPU output checks. Twenty-four rejection cases cover unsupported source constructs, invalid dimensions and accesses, non-finite constants, changed operations, altered loops, and malformed stores. These tests inspect source dependence and rejection behavior in addition to ordinary matrix execution [18].

5.2 Fresh review checks

The review rebuilt the shared body compiler, statement semantics, index proofs, and GPT-2 kernel definitions in an isolated checkout of the cited revision. A focused Lean check proved source equality, static validity, and invocation correctness for the pointwise-add example and audited the shared index, execution, coverage, and disjoint-store theorems. Every printed dependency belonged to the three-axiom allowance. Executable parsing accepted the emitted review shader [19].

The extracted certificate corpus timed out on its first positive example after 90 seconds under the repository’s one-core resource limit. A split diagnostic then checked the token-parser certificate successfully, while the separate lexical certificate exceeded 60 seconds. This identifies lexical certification as one reproduction bottleneck under these limits. The corpus therefore provides no fresh all-cases pass on this machine. The full hybrid session replay and native checkpoint execution were also not repeated during this review. Their reported outcomes above come from the branch’s integration journal, whose generated proof objects and execution reports reside in ignored build directories [19, 16].

5.3 Assurance and reproducibility findings

The examined `Implements` statement excludes the execution model’s failure cases, establishes bounded loop execution, and specifies the output address. The packed connection then identifies those results with the parent byte-array functions. The controller interface assumes shader execution and transfer, while the matrix equivalence follows from checked lemmas. These distinctions support the conditional session claim [4, 8, 9].

The strongest portability limit is the strict arithmetic and host-execution premise. The sampled CPU agreement supports the recorded executions. A browser or physical GPU needs separate evidence for its behavior, and a universal claim would require a proof or enforced implementation of the stated word semantics. The body compiler also needs a refinement theorem and dispatch semantics before its invocation result can be presented as a theorem of a complete formal WGSL execution model [5, 4].

Reproducing the whole result requires rebuilding the generated certificates and preserving their association with the files executed. The source gate checks the parent, generates the hybrid module and shaders, checks wrapper and controller proofs, audits named public theorems, and copies the artifacts into a bundle. The runner subsequently loads bundle files. Its manifest records theorem names and assumptions, but supplies no independent proof that later file changes preserve the checked program [13, 14]. The exact hybrid binary certificate and runtime artifact-identity enforcement are open assurance tasks.

Some status descriptions predate the completed controller proof. The older GPT-2 verification guide still describes full-model composition as pending, and the module-generation CLI prints the same status. The later packed guide, composition gate, and integration journal describe

the conditional complete-session result. Consolidating those descriptions and retaining portable generated certificates and machine-readable test reports would make the branch’s current claim easier to reproduce [15, 16, 13, 18].

6 Reproduction entry points

On a host with the repository’s pinned Lean toolchain and installed native dependencies, the source-build and inference entry points are

```
tools/gpt2-packed build build/gpt2/packed-new
tools/gpt2-packed --prompt 'The purpose of science is' \
  --generate 16 --temperature 0
```

The build destination must be fresh. The source-build tool uses its configured Python environment to prepare checkpoint assets. Inference uses Wasm and WGSL. The packed guide documents bundle selection and generation settings [15, 13]. The review evidence records the focused Lean commands, resource limits, and certificate timeout [19].

References

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