

VQ: Quantum Algorithm Development and Verification with an Evolving Knowledge Base

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Abstract

Quantum algorithm development requires constructions whose behavior and resource costs can be proved, compared, and improved. VQ is a Lean-based system for quantum algorithm development and verification with an evolving knowledge base. Its current focus is elliptic-curve arithmetic and the elliptic-curve discrete logarithm problem. Its goals are to improve algorithms and to improve the knowledge available for developing them. VQ provides circuit and measured-program representations, exact semantics, component specifications, composition theorems, and logical-resource accounting. Its repository comprises an algorithm library, a knowledge base, and experiment records. The library contains formal foundations and constructions from arithmetic through complete algorithms. Checked results include complete point addition, ECDLP recovery probabilities, and logical-resource and Euclidean schedule bounds. The knowledge base holds lemmas, tactics, methods, skills, references, and open questions, with links to their sources and evidence. Experiments preserve the investigations from which both collections develop.

1 Purpose and goals

VQ is a Lean-based system for quantum algorithm development and verification with an evolving knowledge base. Its current focus is quantum elliptic-curve arithmetic and the elliptic-curve discrete logarithm problem (ECDLP). Its mathematical definitions, program representations, proofs, and research records form a common repository for developing and improving algorithms [1].

Algorithm development in this setting involves choices of arithmetic, representation, workspace, measurement, and classical control. These choices affect both correctness and resource costs. For example, an arithmetic component’s meaning includes its treatment of preserved inputs and temporary storage, as well as its numerical output. VQ makes those properties explicit in specifications and connects them to the semantics of the generated quantum program. Resource functions count operations in that same program.

VQ aims to develop algorithms with proved behavior and improved resource bounds, and to improve the knowledge needed to construct, prove, and compare them. A reusable contribution can therefore be a circuit, a theorem, a proof method, an applicability condition, a counterexample, or a research question.

Incremental development determines the organization. A contribution can begin with an idea or a paper to investigate and acquire definitions, proofs, examples, and evidence as work proceeds. A local question can use a selected part of the library and knowledge base. Alternative algorithms and partial candidates can coexist, preserving choices for later comparisons.

A longer-term goal is to support collections of agents exploring algorithm variants and revising shared knowledge. The proposed search would compare candidates under stated input conditions and

resource objectives, retain useful alternatives, and combine compatible results. The present system supplies the source organization, proof interfaces, and versioned knowledge on which such investigations can build.

2 System organization

VQ has three connected collections: the algorithm library, the knowledge base, and experiment records (Table 1). They share one repository and use ordinary Lean source and Markdown documents. The architecture guide defines these collections and their relationships.

Collection	Contents	Relationship to the other collections
Library	Formal foundations, algorithm constructions, proofs, examples, and retained candidates	Supplies the definitions and declarations cited by knowledge entries and studied in experiments
Knowledge base	Results, tactics, methods, skills, references, ideas, and questions	Connects reusable knowledge to library sources and the evidence behind its claims
Experiments	Questions, alternatives, attempts, comparisons, and results	Preserves investigations that produce or revise library and knowledge-base contributions

Table 1: VQ’s repository collections.

The library contains the common formal foundations: quantum program syntax, semantics, register specifications, and resource definitions. Algorithm modules use these foundations and contain their constructions and proofs together. The active source hierarchy is `library/VQ/`, divided into `Quantum`, `Arithmetic`, `Euclid`, `Curve`, and `ECDLP`. Worked examples and retained research candidates have their own directories within `library/`.

The KB guide describes entries under `kb/entries/`. Overlapping guides in `kb/topics/` connect subjects across the source hierarchy. An entry can link to a formal declaration, an experiment, a paper, or another entry. This organization gives a question several routes to relevant knowledge: by mathematical operation, implementation constraint, proof method, or source declaration.

The experiment guide describes records under `experiments/`, grouped by subject and investigation. Source revisions and supporting records connect a result to the inputs from which it arose. Git preserves earlier versions of constructions, entries, and experiments. The root Lake workspace pins Lean and its dependencies and supports imports of individual library modules.

3 Formal foundations

3.1 Program representations and semantics

VQ represents quantum algorithms as Lean data and circuit-generating functions. The circuit language and measured-program language share the gate representation. A generated circuit declares its quantum width and a finite gate list. The gate language includes Hadamard, Pauli, phase, controlled-X, and controlled-controlled-Z (CCZ) operations. Well-formedness predicates check operand bounds, required wire distinctness, and the phase level used by the semantics.

Exact amplitudes at phase level $\ell \geq 3$ belong to the ring

$$D_\ell = (\mathbb{Z}[X]/(X^{2^{\ell-1}} + 1))[1/2].$$

The class of X represents a primitive 2^ℓ th root of unity. The ring contains $1/\sqrt{2}$ and the dyadic phases available at that level. VQ connects this exact algebra to complex-valued quantum semantics and probabilities. Symbolic identities over the circuit generators and state semantics support proofs at cryptographic register widths.

Reversible arithmetic has a separate representation using X, CNOT, and Toffoli gates acting on natural-number basis indices. Register layouts specify which bits encode each field. The reversible compiler replaces a Toffoli with Hadamard–CCZ–Hadamard, and a semantic correspondence theorem relates the compiled quantum action on a basis state to the reversible action. This correspondence lets arithmetic proofs use bit-level and integer reasoning while retaining their meaning in quantum semantics.

Measured programs add measurement, reset, mutable classical bits, immutable classical inputs, and branches controlled by classical bits. Their semantics produces a finite list of execution branches. Each branch contains an outcome record, classical state, and subnormalized quantum state. Its squared norm determines its probability after interpretation in the complex numbers. This representation accounts for intermediate measurement and classically controlled correction within arithmetic, as well as terminal sampling.

3.2 Specifications and composition

VQ’s register specifications describe the behavior of a component on its admitted inputs. A specification contains a width w , an input predicate $A(i)$, and an output relation $R(i, j)$ on basis indices. The relation can state the computed value, preservation of other fields, and restoration of workspace. A reversible implementation meets the specification when its basis action satisfies R for every in-width input satisfying A .

For a measured implementation, the predicate **RealisesAt** also constrains branch amplitudes. At a fixed immutable classical input, it chooses an output map f and an amplitude map α before quantifying over admitted basis inputs. For an input i and terminal branch b , with outcome record r_b and quantum state ψ_b , it requires

$$R(i, f(i)), \quad |\psi_b\rangle = \alpha(r_b)|f(i)\rangle, \quad \sum_b \|\psi_b\|^2 = 1.$$

The same amplitude map applies to every admitted input. The library’s linearity theorem then establishes the corresponding action on two-term superpositions. This input-independent branch factor expresses the coherence needed when measured arithmetic forms part of a larger quantum computation.

The predicate **ImplementsRetainedU** describes measured components that preserve a specified prefix of mutable classical storage. Its parameters identify the quantum domain, reference action, retained bits, and amplitude contribution of newly appended outcomes. Earlier classical results can therefore remain part of a component interface together with its quantum action and the amplitudes of new measurement outcomes.

Composition theorems combine component actions when the first output meets the next input condition and the intermediate register lies within the declared width. Placement theorems transport local constructions into larger registers under range, injectivity, and disjointness hypotheses. These interfaces make input representation, workspace, and preserved state part of the proof that components fit together.

3.3 Logical-resource accounting

VQ computes logical-resource reports from the program syntax to which a correctness theorem refers. Circuit reports include declared width, touched wires, gate counts, and dependency depth. Measured-

program reports also include measurements, resets, mutable classical storage, and immutable input width. A compiled reversible Toffoli contributes one primitive CCZ to the Toffoli-class count.

Static additive costs form intervals. Sequential composition adds costs, and a classical branch takes the minimum of its alternatives’ lower bounds and the maximum of their upper bounds. Equal endpoints give an exact count. The execution-cost semantics also attaches an accumulated cost to each terminal branch.

For a program P , classical input x , and basis input i , let p_b be the probability of terminal branch b and C_b its accumulated cost for a selected additive metric. The expected cost is

$$\mathbb{E}[C \mid x, i] = \sum_b p_b C_b.$$

VQ proves that this expectation is bounded by the static upper endpoint when P is well formed and i lies within its width. A singleton static interval gives equality. A separate definition averages over a specified finite distribution of inputs.

These reports state costs in VQ’s logical program model. Quantum width counts the declared register extent, and dependency depth follows operand dependencies. A resource theorem identifies its program, metric, and hypotheses, so a comparison can express a tradeoff between gate count, allocation, and measurement under the same semantics.

4 Algorithm library

The algorithm library contains both complete algorithms and the components from which they are constructed. Its subject hierarchy follows the mathematical structure of the elliptic-curve applications. Quantum foundations include Fourier transforms, phase estimation, and probability. Arithmetic includes adders, comparisons, multiplication, squaring, constant arithmetic, and lookup. Euclidean constructions supply inversion, coefficient reconstruction, recorded histories, and measured workspace erasure. Curve modules combine field arithmetic into point operations, and ECDLP modules contain scalar computations, Fourier sampling, classical recovery, and whole-program resource proofs.

A construction and its specifications retain the conditions that determine its applicability. These include number representation, admitted input values, clean or borrowed workspace, preserved registers, and allocation. Alternative constructions can realize the same mathematical operation with different conditions and costs. The library’s compact and Karatsuba squaring constructions, for example, have separate allocation and workspace requirements and a proved reference-action comparison. They remain distinct library choices.

Worked examples preserve small applications of component interfaces. Retained candidates preserve alternative constructions and their source identities. Together with the active modules, these collections make earlier algorithm work available for inspection, comparison, and extension.

5 Checked results

VQ contains checked functional, probability, and logical-resource results for secp256k1 point addition and ECDLP, and bounds for arithmetic and classical recovery components. Each result refers to a named program or arithmetic model. Toffoli-class costs count primitive CCZ operations in the compiled program. Phase gates retain their exact semantics at the stated phase level.

5.1 Complete runtime point addition

The wide and compact runtime point-adders implement secp256k1 group addition, including infinity and exceptional cases. Both assume a valid classical addend, a valid encoded affine accumulator,

zeroed quantum workspace, a basis input within the declared width, and phase level $\ell \geq 3$. Their complete theorems establish well-formedness, the intended output with probability one, restored quantum workspace, and the bounds in Table 2. The coherence theorem gives a common branch-amplitude factor for supported target states at a fixed valid addend.

Resource	Wide	Compact
Quantum width w	1,601	1,488
Mutable classical bits	13,342	13,342
Immutable classical input bits	512	512
Toffoli-class upper bound $C_{\max}$	3,070,200	4,561,806
Expected Toffoli-class upper bound	3,070,200	4,561,806
Measurement upper bound	1,959,699	2,413,324
Upper bound on $w \mathbb{E}[C]$	4,915,390,200	6,787,967,328

Table 2: Checked logical resources for the two runtime point-adders. The expectation is over measurement outcomes at a fixed admitted input.

The wide implementation exchanges 113 additional quantum positions for lower Toffoli-class and measurement bounds. Both implementations expose the same 512-bit classical addend and restore the quantum workspace beyond the 512-bit point encoding. These results permit a choice of implementation under an explicit width or gate-cost objective.

5.2 ECDLP recovery and resources

Let G be the secp256k1 generator of order q , and let the valid public point satisfy $Q = dG$ with $0 < d < q$. At phase level $\ell \geq 257$ and from all-zero initialization, the recorded-history complete theorem establishes one-run recovery probability greater than 99/100 for its generated ECDLP program and bounded classical decoder. The theorem gives quantum width 1,488, 1,025 mutable classical bits, exactly 105,732,512 measurements, and static and expected Toffoli-class costs at most 257,008,864. The program incorporates Q during construction and declares zero immutable classical input bits. Its costs cover initialization, both scalar computations, point translation, and quantum cleanup.

The packed-affine family uses an initial table lookup and five-bit scalar windows. Under the same public-point, phase-level, and initialization conditions, its execution and recovery theorems establish restored quantum workspace and recovery probability greater than 99/100. The family has quantum width 833, 768 mutable classical bits, and zero declared immutable input bits. These results have a recorded complete check at source revision `c46c2892`.

For the packed-affine program, let $c \in \{0, \dots, 25\}$ count the five-bit windows absorbed into the initial lookup, whose prefix has $k = 5c + 1$ bits. The resource theorems give an exact public-point-dependent count $C(c, Q)$ shared by every execution path, and hence by the expected cost. They prove the uniform bound

$$C(c, Q) \leq B(c) = 171,630,174 + (102 - c) 172,010,622 + 2^{5c+1} - 2$$

and $B(5) \leq B(c)$ throughout this family. Table 3 gives selected instances and the initial-lookup bounds. The table represents each public point by 512 bits. Its storage is $512 \cdot 2^k$ bits, and the initial lookup uses at most $514 \cdot 2^k - 2$ CNOTs. The minimizing 26-bit prefix therefore has an explicit classical storage and CNOT cost.

The bounded decoder has a separate classical-operation result: 514 scalar-multiplication calls, at most 66,049 candidate comparisons, and at most one modular inversion for each sampled pair. These are counts of decoder operations. The quantum resource bounds and the classical decoder bound describe different parts of the same recovery computation.

Prefix bits k	Whole-program $B(c)$	Initial-lookup CNOT bound	Public table
11	17,372,694,420	1,052,670	128 KiB
16	17,200,747,286	33,685,502	4 MiB
21	17,030,768,280	1,077,936,126	128 MiB
26	16,923,769,370	34,493,956,094	4 GiB
31	18,832,133,532	1,103,806,595,070	128 GiB

Table 3: Checked lookup tradeoffs for the 833-qubit measured ECDLP family. Public-table storage counts packed point data.

5.3 Euclidean termination and schedule bounds

For an odd prime $p < 2^n$ and $0 < x < p$, the recorded binary Euclidean trace reaches $(1, 0)$ within $2n - 1$ steps. The proof uses descent of the sum of the two register bit lengths. At $n = 256$, the bound is 511 steps.

The quotient-based Euclidean model has a different step accounting. For ordinary Euclidean quotients $q_1, \dots, q_D$ starting from $0 < b \leq a$, define

$$W(a, b) = \sum_{j=1}^D \text{bitLength}(q_j), \quad E(a, b) = \sum_{j=1}^D (\text{bitLength}(q_j) - 1).$$

Here $\text{bitLength}(0) = 0$ and $\text{bitLength}(u) = \lfloor \log_2 u \rfloor + 1$ for $u > 0$. The general bounds give $W = E + D$, $E \leq \text{bitLength}(a)$, $D \leq 2 \text{bitLength}(a)$, and $W \leq 3 \text{bitLength}(a)$. For a prime $p < 2^{256}$ and $0 < a < p$, preprocessing uses $x = \min(a, p - a)$. The preprocessed schedule theorem proves $W(p, x) \leq 405$ and the associated round bound $4W(p, x) \leq 1620$. The 511-step and 1,620-round bounds refer to their respective transition systems.

6 Evolving knowledge base

6.1 Knowledge and its sources

The knowledge base (KB) contains reusable knowledge about algorithms and their verification. Entries can contain lemmas, tactics, guidelines, skills, constructions, references, ideas, and unresolved questions. A lemma entry identifies a proposition and its formal source. A tactic or proof-method entry records a way to construct proofs. A guideline records advice and its applicability. A skill gives a procedure an agent can follow. An entry can combine these roles.

An entry may begin with a single idea or a paper reference and a research question. Its directory gives it an identity, and its initial document can be a short Markdown file. Applicability conditions, source links, examples, relationships, and supporting files can develop as the entry’s claims and uses develop. This permits incremental contributions at different stages of research.

Formal source and explanatory knowledge have complementary roles. A Lean declaration states a proposition with its hypotheses. A KB entry explains where the result applies, connects it to alternatives, and links to supporting evidence. An open question can share an entry with established supporting lemmas. Evidence attaches to the individual claim, allowing a topic to contain both proved results and proposed extensions.

Squaring and coefficient reconstruction illustrate the overlapping topic organization. Squaring connects arithmetic identities, alternative constructions, placement, and measured cleanup. Coefficient reconstruction connects Euclidean recurrences, history representations, and normalization questions. These guides link to the relevant source and entries across the library’s subject directories.

6.2 Skills and knowledge revision

VQ includes the procedure for using its KB as a KB entry. The `use-kb` skill describes finding relevant knowledge, inspecting its sources, applying or investigating it, and retaining the result. An agent task can name this skill as its entry point. Subject-specific procedures reside in their corresponding entries, so the shared skill can refer to a component-reuse method without carrying every arithmetic detail.

The skill is versioned with the knowledge it helps an agent consult. Recorded use can supply a correction, a missing search term, a better example, or a more precise applicability condition. Such findings can revise the relevant entry, including the skill. The experiment retains the observation and source revision that support the change.

Knowledge evolution includes revising, splitting, and combining entries as evidence changes. Earlier identities retain links through source history and supporting records. A failed approach can constrain a later proposal, and a counterexample can identify the condition under which a claim fails. The KB thereby preserves both reusable results and the reasons for revising them.

7 Experiment records and agent development

An experiment is a record of a particular investigation. Its contents identify the question, selected source and KB revisions, changes, attempts, results, and supporting evidence. The record can concern an algorithm, a proof method, a paper assessment, or the KB’s organization. One experiment can revise several entries, and one entry can accumulate evidence from several experiments.

This organization connects algorithm development to knowledge development. A comparison can produce a construction and a theorem, while also clarifying the conditions under which an earlier result applies. A research question can acquire a partial proof and a narrower remaining question. The experiment preserves that particular investigation, and the library and KB make its reusable contributions available to later work.

For agent investigations, task identity includes the source and knowledge revisions available to the agent. Separate writable worktrees and build directories preserve the inputs of concurrent variants. A retained comparison identifies the candidate changes, assumptions, resource model, and proof results. These records supply the basis for comparing agent contributions and for attributing a knowledge revision to the evidence that produced it.

The longer-term search goal applies to both collections. Algorithm variants can differ in arithmetic, allocation, or control structure. Knowledge variants can differ in an entry’s explanation, examples, organization, or procedure. Their evaluation needs corresponding criteria: proved program behavior and resource bounds for algorithms, and evidence about finding and applying knowledge for KB changes. VQ’s versioned sources and experiment records provide inputs for those comparisons.

8 Related work

VQ combines functions that other systems provide through quantum programming languages, proof environments, circuit optimizers, resource analyzers, and algorithm libraries. Table 4 compares the objects and methods of those systems. The comparison uses the versions described in the cited publications. A proof about a program family, an equivalence check between two circuits, and a resource estimate from a component decomposition establish different properties.

Table 4: Systems related to VQ. The last column identifies the object of verification or analysis in the cited work.

System	Representation	Method	Result or analysis
QWIRE [2]	Linear circuit language embedded in a classical host	Operational semantics and density-matrix denotation	Type safety, normalization, and semantic preservation of circuit reduction
SQIR/VOQC [3, 4]	Indexed quantum circuits embedded in Coq	Symbolic matrix reasoning and proved transformations	Semantics-preserving optimization. SQIR also supports a certified factoring implementation
CoqQ [5]	Quantum programs and operator assertions in Coq	Program logic proved sound against denotational semantics	Compositional functional proofs with local and parallel reasoning
Qbricks [6]	Circuit-generating programs and path-sum specifications	Deductive verification through Why3	Parameterized circuit correctness and resource properties
Qafny [7]	Typed quantum programs and array-based assertions	Translation to Dafny using a separation-logic proof system	Automated functional verification of specified quantum programs
AutoQ 2.0 [8]	Sets of pure states represented by tree automata	Automata operations and SMT-assisted entailment	Partial correctness of programs with measurement, branching, and annotated loops
CertiQ [9]	Compiler passes over circuit representations	Circuit calculus and SMT verification conditions	Equivalence-preserving implementations of Qiskit compiler passes
PyZX [10]	ZX-diagrams representing linear maps	Graph rewriting and circuit extraction	Circuit optimization and equality validation
QuRA [11]	Circuit-generating programs with indexed types	Metric-parameterized type-and-effect analysis	Symbolic bounds on width, gate counts, and depth
Qet [12]	Mixed classical-quantum programs with cost annotations	Expectation transformers and constraint solving	Upper bounds on expected execution cost, including measured loops
Qualtran [13]	Typed algorithm components, decompositions, and call graphs	Composition, symbolic counting, and simulation protocols	Reusable algorithm library, logical counts, and physical-resource estimates under specified models
LeanDojo [15]	Lean proof states, tactics, and available premises	Program analysis and retrieval-augmented proof search	Proof-environment interaction, premise selection, and tactic generation
VQ	Lean circuit generators, measured programs, and linked KB entries	Semantic proofs, component composition, and program-based cost accounting	Quantum EC constructions with functional, recovery, and resource theorems, plus versioned research knowledge

8.1 Quantum languages and proof environments

QWIRE [2] embeds a linearly typed circuit language in a classical host. Its separation between host computation and quantum data supports circuit generation, measurement, and the use of measured values in the host. The language has a density-matrix denotation and operational soundness results, with type safety and normalization conditional on the corresponding host properties. QWIRE and VQ both give generated circuits a mathematical meaning and use interfaces to control composition. QWIRE expresses wire-use discipline through linear typing. VQ’s arithmetic interfaces add value-level conditions on encoded registers, preserved fields, and workspace restoration.

SQIR and VOQC [3] combine an indexed circuit representation with Coq proofs of optimizer correctness. SQIR assigns matrices to circuits, and VOQC proves that its transformations preserve that denotation. The verified optimizer extracts to executable OCaml. This organization makes a proved transformation reusable across circuits. VQ’s arithmetic component theorems identify the computed operation and its admitted register states.

The SQIR-based Shor development [4] extends the comparison to a complete algorithm. Peng et al. verify the quantum and classical parts of an integer-factoring implementation in Coq, including the connection between quantum sampling and successful classical recovery. Its reversible intermediate representation separates proofs of classical arithmetic from their quantum interpretation, as VQ’s reversible language and compiler correspondence do. Both the factoring development and VQ’s ECDLP proofs connect implemented arithmetic to a probabilistic application result, under specifications of the input domain, sampling procedure, and recovery criterion.

CoqQ [5] supplies a quantum programming language and a program logic whose soundness is proved in Coq over Mathematical Components and its analysis library. Assertions can use Dirac expressions, and its reasoning rules support local and parallel composition. CoqQ organizes proofs around its assertion logic. VQ’s measured-arithmetic interfaces expose outcome records, retained classical state, and input-independent branch amplitudes. Those interface choices determine the hypotheses a caller must establish.

Qbricks [6] verifies circuit-building programs with parameterized path-sum specifications and a deductive system implemented through Why3. Its examples include phase estimation, Grover search, and the quantum order-finding part of Shor’s algorithm. A specification can describe a family of generated circuits and support proofs about both its action and its resources. This corresponds to VQ’s use of circuit-generating definitions at symbolic widths. The proof methods differ: Qbricks translates its specifications into deductive verification conditions, whereas VQ expresses the generator, semantics, and proof as Lean declarations.

Qafny [7] translates typed quantum programs and their specifications into Dafny. Its proof system represents quantum operations through classical array operations and uses separation logic to organize reasoning about state. The paper proves soundness and completeness of the proof system and demonstrates verification on quantum walks, Grover search, and Shor’s algorithm. VQ provides reusable Lean lemmas and proof procedures whose applications produce proof terms checked in the library’s formal environment.

AutoQ 2.0 [8] represents sets of pure quantum states with level-synchronized tree automata. It combines operations on those automata with SMT-assisted entailment checking to verify programs containing measurement and classical control. Users supply preconditions, postconditions, and loop invariants. Its partial-correctness judgments compare states up to a nonzero complex factor. VQ’s branch semantics retains amplitudes and their probability interpretation because its application theorems include recovery probability and expected cost. The two specification choices support different conclusions about a measured execution.

8.2 Circuit transformation systems

CertiQ [9] targets the correctness of compiler passes, including the circuit data structures and representation conversions those passes use. Its circuit calculus turns equivalence obligations into SMT problems, and its development produces verified implementations of Qiskit passes. The reusable object is therefore a transformation with assumptions on its input representation. VQ expresses an arithmetic substitution through a specification of the computed value, register preservation, and workspace restoration. Both forms of verification require an explicit relation between the input and output representations.

PyZX [10] provides optimization, equality validation, and visualization through the ZX-calculus. It rewrites graph representations of linear maps and extracts circuits from the resulting diagrams. VOQC proves transformations over SQIR circuit syntax. VQ’s retained arithmetic alternatives can change the decomposition of an operation, the workspace allocation, and the use of measurement. A comparison with either optimizer therefore depends on the equivalence relation and admitted inputs of the particular transformation.

8.3 Resource analysis and algorithm libraries

QuRA [11] uses indexed types and effects to describe the resources of circuit-generating programs. Its analysis is parameterized by a metric, supporting width, gate count, depth, and restrictions to selected wire or gate kinds. The paper establishes soundness under conditions on the metric interpretation. QuRA obtains symbolic bounds through typing. VQ’s resource theorems connect functions of program syntax to execution costs. The point-adder and lookup results in Section 5 instantiate those functions for specified constructions.

Qet [12] analyzes expected cost for programs with intermediate measurement, classical control, and loops. It expresses the analysis through quantum expectation transformers, generates constraints for loop bounds, and reduces the resulting conditions to solver queries. Its examples include repeat-until-success computations. Qet’s analysis can infer a bound dependent on the input state. VQ defines expectation from the probabilities and costs of its finite execution branches and proves bounds, including equality when every branch has the same cost. This distinction identifies both the program class and the method by which an expected-cost claim is obtained.

Qualtran [13] organizes algorithm development around Python components, called bloqs, that have typed register signatures. Decompositions, call graphs, simulation behavior, and resource annotations can develop independently for each component. Hierarchical call graphs support symbolic counting without expanding every gate. The library includes arithmetic and elliptic-curve discrete-logarithm constructions, as well as simulation and chemistry algorithms. Its logical-resource analyses can feed specified physical-resource models. VQ associates its constructions with Lean specifications and theorems, including workspace and branch-amplitude conditions, and links reusable explanations and alternatives through its KB.

The assurance associated with a resource number depends on its source. A type-derived bound, a count computed from a decomposition, a probability-weighted execution cost, and a physical-resource estimate each refer to a defined model. For the VQ results reported here, the counted object is the generated logical program. Comparisons with another system therefore depend on its gate basis, allocation policy, measurement model, and treatment of classical work.

8.4 Reusable proofs and research knowledge

The Lean mathematical library, mathlib [14], provides reusable definitions, theorems, and tactics organized through Lean’s module and dependency structure. VQ builds its domain-specific formal library in that setting. Its KB records additional information associated with a result: application conditions,

comparisons between constructions, research questions, and procedures for using the available proofs. A formal declaration gives the proposition and its proof dependencies. A linked entry explains its use in algorithm development.

LeanDojo [15] provides programmatic interaction with Lean and extracts proof data, including the premises used by tactics. Its ReProver system retrieves accessible premises to support language-model proof search. VQ’s KB-use skill specifies a research task: finding relevant entries, checking their formal sources, applying a result or investigating a question, and recording what the investigation changes. The proposed multi-agent algorithm search would consume both formal declarations and these research records.

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