

VQ: Quantum Algorithm Development and Verification with an Evolving Knowledge Base

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Abstract

Quantum algorithm development requires constructions whose behavior and resource costs can be proved, compared, and improved. VQ is a Lean-based system for quantum algorithm development and verification with an evolving knowledge base. Its current focus is elliptic-curve arithmetic and the elliptic-curve discrete logarithm problem. Its goals are to improve algorithms and to improve the knowledge available for developing them. VQ provides circuit and measured-program representations, exact semantics, component specifications, composition theorems, and logical-resource accounting. Its repository comprises an algorithm library, a knowledge base, and experiment records. The library contains formal foundations and constructions from arithmetic through complete algorithms. The knowledge base holds lemmas, tactics, methods, skills, references, and open questions, with links to their sources and evidence. Experiments preserve the investigations from which both collections develop.

1 Purpose and goals

VQ is a Lean-based system for quantum algorithm development and verification with an evolving knowledge base. Its current focus is quantum elliptic-curve arithmetic and the elliptic-curve discrete logarithm problem (ECDLP). Its mathematical definitions, program representations, proofs, and research records form a common repository for developing and improving algorithms [1, 2].

Algorithm development in this setting involves choices of arithmetic, representation, workspace, measurement, and classical control. These choices affect both correctness and resource costs. For example, an arithmetic component’s meaning includes its treatment of preserved inputs and temporary storage, as well as its numerical output. VQ makes those properties explicit in specifications and connects them to the semantics of the generated quantum program. Resource functions count operations in that same program [9, 12].

VQ aims to develop algorithms with proved behavior and improved resource bounds, and to improve the knowledge needed to construct, prove, and compare them. A reusable contribution can therefore be a circuit, a theorem, a proof method, an applicability condition, a counterexample, or a research question [2, 4].

Incremental development determines the organization. A contribution can begin with an idea or a paper to investigate and acquire definitions, proofs, examples, and evidence as work proceeds. A local question can use a selected part of the library and knowledge base. Alternative algorithms and partial candidates can coexist, preserving choices for later comparisons [3, 4, 5].

A longer-term goal is to support collections of agents exploring algorithm variants and revising shared knowledge. The proposed search would compare candidates under stated input conditions and resource objectives, retain useful alternatives, and combine compatible results. The present system

supplies the source organization, proof interfaces, and versioned knowledge on which such investigations can build [2].

2 System organization

VQ has three connected collections: the algorithm library, the knowledge base, and experiment records (Table 1). They share one repository and use ordinary Lean source and Markdown documents [2].

Collection	Contents	Relationship to the other collections
Library	Formal foundations, algorithm constructions, proofs, examples, and retained candidates	Supplies the definitions and declarations cited by knowledge entries and studied in experiments
Knowledge base	Results, tactics, methods, skills, references, ideas, and questions	Connects reusable knowledge to library sources and the evidence behind its claims
Experiments	Questions, alternatives, attempts, comparisons, and results	Preserves investigations that produce or revise library and knowledge-base contributions

Table 1: VQ’s repository collections.

The library contains the common formal foundations: quantum program syntax, semantics, register specifications, and resource definitions. Algorithm modules use these foundations and contain their constructions and proofs together. The active source hierarchy is `library/VQ/`, divided into `Quantum`, `Arithmetic`, `Euclid`, `Curve`, and `ECDLP`. Worked examples and retained research candidates have their own directories within `library/` [3].

Knowledge entries reside under `kb/entries/`. Overlapping guides in `kb/topics/` connect subjects across the source hierarchy. An entry can link to a formal declaration, an experiment, a paper, or another entry. This organization gives a question several routes to relevant knowledge: by mathematical operation, implementation constraint, proof method, or source declaration [4].

Experiments reside under `experiments/`, grouped by subject and investigation. Source revisions and supporting records connect a result to the inputs from which it arose. Git preserves earlier versions of constructions, entries, and experiments. The root Lake workspace pins Lean and its dependencies and supports imports of individual library modules [2, 5].

3 Formal foundations

3.1 Program representations and semantics

VQ represents quantum algorithms as Lean data and circuit-generating functions. A generated circuit declares its quantum width and a finite gate list. The gate language includes Hadamard, Pauli, phase, controlled-X, and controlled-controlled-Z (CCZ) operations. Well-formedness predicates check operand bounds, required wire distinctness, and the phase level used by the semantics [6].

Exact amplitudes at phase level $\ell \geq 3$ belong to the ring

$$D_\ell = (\mathbb{Z}[X]/(X^{2^{\ell-1}} + 1))[1/2].$$

The class of X represents a primitive 2^ℓ th root of unity. The ring contains $1/\sqrt{2}$ and the dyadic phases available at that level. VQ connects this exact algebra to complex-valued quantum semantics

and probabilities. Symbolic identities over the circuit generators and state semantics support proofs at cryptographic register widths [6, 7].

Reversible arithmetic has a separate representation using X, CNOT, and Toffoli gates acting on natural-number basis indices. Register layouts specify which bits encode each field. The reversible compiler replaces a Toffoli with Hadamard–CCZ–Hadamard, and a semantic correspondence theorem relates the compiled quantum action on a basis state to the reversible action. This correspondence lets arithmetic proofs use bit-level and integer reasoning while retaining their meaning in quantum semantics [8].

Measured programs add measurement, reset, mutable classical bits, immutable classical inputs, and branches controlled by classical bits. Their semantics produces a finite list of execution branches. Each branch contains an outcome record, classical state, and subnormalized quantum state. Its squared norm determines its probability after interpretation in the complex numbers. This representation accounts for intermediate measurement and classically controlled correction within arithmetic, as well as terminal sampling [6, 7].

3.2 Specifications and composition

VQ’s register specifications describe the behavior of a component on its admitted inputs. A specification contains a width w , an input predicate $A(i)$, and an output relation $R(i, j)$ on basis indices. The relation can state the computed value, preservation of other fields, and restoration of workspace. A reversible implementation meets the specification when its basis action satisfies R for every in-width input satisfying A [9].

For a measured implementation, the predicate **RealisesAt** also constrains branch amplitudes. At a fixed immutable classical input, it chooses an output map f and an amplitude map α before quantifying over admitted basis inputs. For an input i and terminal branch b , with outcome record r_b and quantum state ψ_b , it requires

$$R(i, f(i)), \quad |\psi_b\rangle = \alpha(r_b)|f(i)\rangle, \quad \sum_b \|\psi_b\|^2 = 1.$$

The same amplitude map applies to every admitted input. The library’s linearity theorem then establishes the corresponding action on two-term superpositions. This input-independent branch factor expresses the coherence needed when measured arithmetic forms part of a larger quantum computation [9].

The predicate **ImplementsRetainedU** describes measured components that preserve a specified prefix of mutable classical storage. Its parameters identify the quantum domain, reference action, retained bits, and amplitude contribution of newly appended outcomes. Earlier classical results can therefore remain part of a component interface together with its quantum action and the amplitudes of new measurement outcomes [10].

Composition theorems combine component actions when the first output meets the next input condition and the intermediate register lies within the declared width. Placement theorems transport local constructions into larger registers under range, injectivity, and disjointness hypotheses. These interfaces make input representation, workspace, and preserved state part of the proof that components fit together [10, 11].

3.3 Logical-resource accounting

VQ computes logical-resource reports from the program syntax to which a correctness theorem refers. Circuit reports include declared width, touched wires, gate counts, and dependency depth. Measured-program reports also include measurements, resets, mutable classical storage, and immutable input width. A compiled reversible Toffoli contributes one primitive CCZ to the Toffoli-class count [12, 8].

Static additive costs form intervals. Sequential composition adds costs, and a classical branch takes the minimum of its alternatives’ lower bounds and the maximum of their upper bounds. Equal endpoints give an exact count. The execution-cost semantics also attaches an accumulated cost to each terminal branch [12].

For a program P , classical input x , and basis input i , let p_b be the probability of terminal branch b and C_b its accumulated cost for a selected additive metric. The expected cost is

$$\mathbb{E}[C \mid x, i] = \sum_b p_b C_b.$$

VQ proves that this expectation is bounded by the static upper endpoint when P is well formed and i lies within its width. A singleton static interval gives equality. A separate definition averages over a specified finite distribution of inputs [13].

These reports state costs in VQ’s logical program model. Quantum width counts the declared register extent, and dependency depth follows operand dependencies. A resource theorem identifies its program, metric, and hypotheses, so a comparison can express a tradeoff between gate count, allocation, and measurement under the same semantics [12].

4 Algorithm library

The algorithm library contains both complete algorithms and the components from which they are constructed. Its subject hierarchy follows the mathematical structure of the elliptic-curve applications. Quantum foundations include Fourier transforms, phase estimation, and probability. Arithmetic includes adders, comparisons, multiplication, squaring, constant arithmetic, and lookup. Euclidean constructions supply inversion, coefficient reconstruction, recorded histories, and measured workspace erasure. Curve modules combine field arithmetic into point operations, and ECDLP modules contain scalar computations, Fourier sampling, classical recovery, and whole-program resource proofs [3, 14].

A construction and its specifications retain the conditions that determine its applicability. These include number representation, admitted input values, clean or borrowed workspace, preserved registers, and allocation. Alternative constructions can realize the same mathematical operation with different conditions and costs. The library’s compact and Karatsuba squaring constructions, for example, have separate allocation and workspace requirements and a proved reference-action comparison. They remain distinct library choices [15].

Complete algorithm declarations connect these component results to an application-level statement. The runtime `secp256k1` point-adder accepts a classical addend and acts on an affine quantum accumulator. Its valid-point domain covers infinity and exceptional group-law cases. The complete result assumes a valid addend, a valid encoded accumulator point, zeroed quantum workspace, an input within the declared width, and phase level at least three. It connects the generated program to group addition, restored quantum workspace, and resource bounds. Its superposition theorem gives a common branch amplitude for supported target states at a fixed valid addend [16].

The recorded-history ECDLP construction combines point arithmetic, semiclassical Fourier sampling, and a bounded classical decoder. For the `secp256k1` generator G of order q and a valid public point $Q = dG$ with $0 < d < q$, its complete theorem proves one-run recovery probability greater than 99/100 from zero initialization at phase level at least 257. The same theorem states well-formedness, register sizes, measurement count, and upper bounds on static and expected Toffoli-class cost. This is an example of VQ’s complete-algorithm specification: semantic and resource claims refer to the same generated program [17].

Worked examples preserve small applications of component interfaces. Retained candidates preserve alternative constructions and their source identities. Together with the active modules, these collections make earlier algorithm work available for inspection, comparison, and extension [3].

5 Evolving knowledge base

5.1 Knowledge and its sources

The knowledge base (KB) contains reusable knowledge about algorithms and their verification. Entries can contain lemmas, tactics, guidelines, skills, constructions, references, ideas, and unresolved questions. A lemma entry identifies a proposition and its formal source. A tactic or proof-method entry records a way to construct proofs. A guideline records advice and its applicability. A skill gives a procedure an agent can follow. An entry can combine these roles [4].

An entry may begin with a single idea or a paper reference and a research question. Its directory gives it an identity, and its initial document can be a short Markdown file. Applicability conditions, source links, examples, relationships, and supporting files can develop as the entry’s claims and uses develop. This permits incremental contributions at different stages of research [4].

Formal source and explanatory knowledge have complementary roles. A Lean declaration states a proposition with its hypotheses. A KB entry explains where the result applies, connects it to alternatives, and links to supporting evidence. An open question can share an entry with established supporting lemmas. Evidence attaches to the individual claim, allowing a topic to contain both proved results and proposed extensions [4].

Topic guides provide overlapping organization. Squaring connects arithmetic identities, alternative constructions, placement, and measured cleanup. Coefficient reconstruction connects Euclidean recurrences, history representations, and normalization questions. These guides link to the relevant source and entries across the library’s subject directories [14, 15].

5.2 Skills and knowledge revision

VQ includes the procedure for using its KB as a KB entry. The `use-kb` skill describes finding relevant knowledge, inspecting its sources, applying or investigating it, and retaining the result. An agent task can name this skill as its entry point. Subject-specific procedures reside in their corresponding entries, so the shared skill can refer to a component-reuse method without carrying every arithmetic detail [4].

The skill is versioned with the knowledge it helps an agent consult. Recorded use can supply a correction, a missing search term, a better example, or a more precise applicability condition. Such findings can revise the relevant entry, including the skill. The experiment retains the observation and source revision that support the change [4, 5].

Knowledge evolution includes revising, splitting, and combining entries as evidence changes. Earlier identities retain links through source history and supporting records. A failed approach can constrain a later proposal, and a counterexample can identify the condition under which a claim fails. The KB thereby preserves both reusable results and the reasons for revising them [4].

6 Experiment records and agent development

An experiment is a record of a particular investigation. Its contents identify the question, selected source and KB revisions, changes, attempts, results, and supporting evidence. The record can concern an algorithm, a proof method, a paper assessment, or the KB’s organization. One experiment can revise several entries, and one entry can accumulate evidence from several experiments [5].

This organization connects algorithm development to knowledge development. A comparison can produce a construction and a theorem, while also clarifying the conditions under which an earlier result applies. A research question can acquire a partial proof and a narrower remaining question. The experiment preserves that particular investigation, and the library and KB make its reusable contributions available to later work [2, 5].

For agent investigations, task identity includes the source and knowledge revisions available to the agent. Separate writable worktrees and build directories preserve the inputs of concurrent variants. A retained comparison identifies the candidate changes, assumptions, resource model, and proof results. These records supply the basis for comparing agent contributions and for attributing a knowledge revision to the evidence that produced it [2, 5].

The longer-term search goal applies to both collections. Algorithm variants can differ in arithmetic, allocation, or control structure. Knowledge variants can differ in an entry’s explanation, examples, organization, or procedure. Their evaluation needs corresponding criteria: proved program behavior and resource bounds for algorithms, and evidence about finding and applying knowledge for KB changes. VQ’s versioned sources and experiment records provide inputs for those comparisons [2, 4, 5].

7 Related work

VQ shares the use of explicit program semantics and machine-checked proofs with other quantum-verification systems. SQIR and VOQC represent quantum circuits in Coq and prove transformations using symbolic complex-matrix semantics [18]. CoqQ supplies a quantum program logic whose soundness is proved against denotational semantics [19]. Qbricks supplies deductive verification for parameterized circuit-building programs [20]. VQ applies explicit semantics and compositional proofs to elliptic-curve constructions, with knowledge entries and experiment records that connect reusable methods to their sources.

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