

A Formal, Statistical, and Source Audit of Schrottenloher’s Optimized Point-Addition Circuits for Elliptic-Curve Discrete Logarithms

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Abstract

Schrottenloher proposes quantum point-addition circuits whose resource savings depend on a compressed binary-Euclidean trace, reversed Bézout reconstruction, and approximate modular arithmetic. The audit covers arXiv:2606.02235v1 through Section 2, Algorithms 1–11, Figure 1, and Tables 1–3 using Lean 4 proofs, exact arithmetic, statistical records, and executions of pinned publication source. The audit verifies component theorems for the compressed trace, reconstruction, conditional multiplication, and modular arithmetic, gives checked counterexamples to 402-round totality and shrinking-field value preservation, identifies resource discrepancies, and leaves Schrottenloher’s Table 1 aggregate failure probability and whole-circuit ECDLP resources unresolved. The schedule analysis finds a secp256k1 residue whose trace exceeds the 402-round schedule and terminates at round 405, proves the tight 511-round universal bound for the full-width trace, verifies a deterministic 512-round multiplier schedule, and gives a round-369 witness against the shrinking-field value model. Under the reference law, the probability of a terminal input with an approximate-reconstruction failure at one of the 402 reverse indices is below 2^{-29} , and the fixed multiplier’s out-of-domain probability is below its 402-round termination tail plus 2^{-29} . No theorem transfers that law to the paper’s point-addition call states or combines the fixed-model result with the shrinking-field and truncated-comparison failures. The grouped counts derived from the pinned source reproduce all four displayed baseline non-Clifford exponents in Schrottenloher’s Table 1. Captured peak widths are one qubit below two printed widths. Static arithmetic over the pinned full-window source yields a 201,410-unit surcharge, exceeding the paper’s 196,608 by 4,802. Exact source arithmetic corrects two displayed base-two exponents in Schrottenloher’s Table 2 by 0.01. Under the audit-inferred map for Schrottenloher’s Table 3, generic modular squaring rounds to 11 percent. The table prints 10 percent.

1 Audited claims, models, and evidence rules

1.1 Audited construction and dependencies

The audit verifies component results for the compressed Euclidean trace, reconstruction, conditional multiplication, and modular arithmetic. Checked witnesses refute 402-round totality and shrinking-field value preservation. Source reconstruction and captured executions identify discrepancies across Schrottenloher’s Tables 1–3. The Table 1 aggregate failure probability and whole-circuit ECDLP resources remain unresolved.

The audited object is Schrottenloher’s arXiv:2606.02235v1 revision [10]. The audit covers Section 2, Algorithms 1–11, Figure 1, and Tables 1–3, organized as sixteen claim groups. Their dependency order is:

ECDLP recovery $\longrightarrow$ windowed scalar multiplication $\longrightarrow$ Algorithm 1 point addition
 $\longrightarrow$ Algorithm 4 multiplication $\longrightarrow$ Algorithms 2–3 and Figure 1
 $\longrightarrow$ Algorithms 5–11.

The paper’s quantum reduction evaluates two scalar multiplications in superposition and then applies quantum Fourier transforms (QFTs). Any missing premise in the arithmetic oracle limits downstream recovery and composite-resource claims that use that oracle.

Arithmetic value, reversible-circuit action, work- and record-register cleanup, failure probability under a named law, and support for a publication resource count require separate evidence. A *formal result* is a proposition checked by Lean at the cited revision. A *conditional result* retains explicit domain, success, or probability-law premises that

the paper execution has not discharged. A *checked counterexample* is a Lean-checked witness against either a paper claim or an attempted audit lemma, with the target identified. A *source reconstruction* is exact arithmetic over quantities or formulas transcribed from pinned publication source, and a *source execution* is recorded output from that source under a named environment. A *statistical result* depends on its stated sampling model and provenance. The term *unresolved* means that the paper, cited sources, retained code, and audit results do not contain the evidence needed for the claim.

1.2 Parameters and counting convention

The fixed field modulus is the secp256k1 prime

$$p = 2^{256} - 4294968273,$$

with field width $n = 256$ and scalar-window width $w = 16$. Schrottenloher writes q for this modulus. The formal records use p . The windowed construction treats two 256-bit scalars, giving 32 window ordinals before schedule adjustments.

Schrottenloher’s tables group CCX, CCZ, and forward temporary-AND operations as one non-Clifford metric. In the retained Qarton captures, `AndFanoutResourceReporter.ccx_count()` equals raw CCX plus forward `AndGate`, and every capture has raw CCZ count zero. `AndGateUncompute` appears separately and contributes zero to the grouped total.

The external million-residue audit assumes independent and identically distributed (IID) samples. Calls inside one reversible computation may share state. The formal single-transition and single-operation results apply under arbitrary dependence across calls.

VQ denotes the Lean evidence repository at revision 00ce72179927 [13]. The cited statements interpret reversible gates on basis-state bit registers and prove exact register action under their displayed premises. Separate statements prove cleanup, finite-probability bounds, and resource arithmetic. The aggregate build type-checks their paper-facing test modules at the pinned revision. A theorem must connect circuit action, cleanup, probability, and resource arithmetic before a composite claim follows.

1.3 Sources, records, and probability law

The publication evidence consists of the paper, companion revision 3b60d5050b18 [8], schedule-source revision d8d2473cb43c [9], and Qarton revision 7506faba2147 [7]. Formal statements, theorem tests, resource arithmetic, counterexamples, and statistical record checks come from VQ revision 00ce72179927 [12, 11]. The external comparison uses Babbush et al. v2 [2], its public Zenodo deposit with program, outputs, verification keys, and Groth16 proofs [1], and the original Clopper–Pearson interval construction [4]. Appendix D records the statistical hashes and execution boundary.

Retained VQ paths at revision 00ce72179927:

Captured resource directories:

`paper/schrottenloher-resource-capture/schema-v2-baseline`

`paper/schrottenloher-resource-capture/schema-v2-full-window`

Source-analysis reports:

`paper/schrottenloher-402-source-audit.md`

`paper/schrottenloher-resource-audit.md`

Statistical files:

`paper/schrottenloher-tail-statistical-audit.md`

`paper/schrottenloher-tail-statistical-audit-result.md`

`paper/schrottenloher-tail-statistical-audit-manifest.json`

`paper/schrottenloher-tail-statistical-audit-inference.json`

`paper/schrottenloher-tail-statistical-audit-failures.json`

The *reference law* draws the Euclidean input uniformly from $\{1, \dots, p - 1\}$ and, independently, draws the reconstruction coefficient uniformly modulo p . The formal records call this distribution the *reference-product law*, shortened here to *reference law*. Applying a probability theorem under this law to the paper’s basis-label registers requires a transfer theorem or event-specific comparison.

An *accepted nonzero residue* is a 32-byte big-endian candidate interpreted as an integer x with $1 \leq x < p$. A *valid-block readout* reads a valid raw six-bit triple after forward compression as its compressed value below 2^5 , with the sixth wire clear. The reverse action decodes that value. A *captured root event* is the five-tuple recorded when the child class `IPModMul_inv` is appended to the root `WindowAffAddSpaceOpt`: old peak, new current, child peak, old current, and the number of previous qubits inside the mapped child. Qarton’s peak-width recurrence at revision 7506faba2147 is

$$\max(\text{oldPeak}, \text{newCurrent}, \text{childPeak} + \text{oldCurrent} - \text{previousInside}). \quad (1)$$

The retained resource records apply this recurrence to a captured event. The retained Python produces the recorded event trace. The formal calculation takes that trace as its input.

1.4 Full-width, shrinking, approximate, and corrected models

The full-width mathematical trace supplies the transition and termination reasoning without field clipping. The publication-source shrinking schedule uses 402 rounds, padding 37, comparison-truncation parameter 40, and field width 374. Its checked comparison width is the smaller of the active field width and 77. The checked 402-round pseudo-Mersenne multiplier uses padding 40, retains 40 comparison bits, and uses equality-aware addition in the first 50 forward rounds. The deterministic corrected multiplier fixes 512 rounds and enlarges the record layout.

Table 1: Models and checked evidence.

Model	Value and circuit semantics	Cleanup	Termination or probability	Resource evidence
Full-width mathematical trace	Parameterized step, trace, compression, and reconstruction semantics	Trace reversal and reconstruction on the stated domain	The universal bound is 511 rounds and is attained.	The compressor-block count is exact.
Publication shrinking schedule	Conditional schedule-circuit action and every compressed record block under trace-domain, padding, odd-left, and initialization premises	The theorem proves raw/work cleanup under the same premises. Its scope ends at schedule-circuit action.	A checked round-369 witness violates the trace domain. One witness gives a uniform-nonzero lower bound.	Pinned schedule-source formulas supply the resource parameters.
Checked 402 pseudo-Mersenne multiplier	Exact basis-state multiplier action on trace, padding, and approximate-arithmetic success premises	The theorem proves cleanup on that success domain.	A checked 405-round witness refutes universal termination. Under the reference law, reconstruction domain failure is below 2^{-29} , and multiplier domain failure is below the termination tail plus 2^{-29} .	Formal counts cover isolated components. Table 1 composition remains open.
Deterministic 512 multiplier	Corrected fixed-schedule multiplication action with the exact adder	The theorem proves cleanup for every nonzero input and reduced target under its clean-work and register premises.	The tight 511-round theorem gives universal termination.	The record layout is checked. Whole point-addition cost remains open.

The models share the full-width binary-Euclidean transition when all retained values fit and the selected approximate operations satisfy their success predicates. Their schedules, padding, active widths, comparison truncation, equality-aware prefixes, and selected adders differ. A transfer between rows of Table 1 requires a checked theorem.

1.5 Claim-group disposition

Table 2: Disposition of the audited paper claims.

Item	Paper-facing statement	Disposition	Corrected statement or unresolved premise	Sec.
§2	One observation recovers the ECDLP with probability greater than one half.	Formal result and unresolved	A separately generated exact VQ reference circuit is checked through its emitted state, marginal, selected-pair recovery, and resources. A theorem relating Schrottenloher’s Algorithm 1 oracle and recovery law to that circuit is absent.	2.1
Alg. 1	Windowed affine point addition supplies the oracle arithmetic.	Conditional result	The register path and affine-result formula are checked. Separate enabled and disabled value lemmas retain group-validity and noncollision premises or zero-selected-point and representability premises. Exceptional paths, concrete tables, persistent selection state, a measured-unlookup realization, composite resources, and the call-state law are absent.	2.3
Alg. 2	A fixed 402-round binary-Euclidean schedule records the choices and is used with the asserted high-probability bound.	Formal result, checked counterexample, and unresolved	Parameterized trace semantics are checked. One residue disproves total 402-round termination. The claimed tail remains unproved. Every nonzero input terminates by 511 rounds, and that bound is tight.	3.1, 3.5–3.6, 5.2–5.3
Fig. 1	Three Euclidean choices compress reversibly from six bits to five retained bits.	Formal result	Forward and inverse actions, valid-block readout, and the 13-gate cost are checked. Reconstruction’s decoder handles a partial final block.	3.2–3.3
Alg. 3	Reversing the choices reconstructs the Bézout coefficient.	Formal result and conditional result	Pure compressed reconstruction is checked from a terminal trace. The generated approximate reverse schedule separately requires successful selected coefficient steps.	3.3
Alg. 4	Trace and reconstruction form an in-place modular multiplier.	Conditional result, formal result, and checked counterexamples	The checked 402-round pseudo-Mersenne multiplier is conditional on its success domain. Under the reference law, the probability of a terminal input with an approximate-reconstruction failure at one of the 402 reverse indices is below 2^{-29} , and multiplier domain failure is below the termination tail plus 2^{-29} . A fixed 512-round version gives deterministic termination with an 855-bit record. The publication shrinking schedule has a round-369 value failure. Stage refinement, exact value action, and work-register cleanup are checked for odd modulus and stated width premises.	3.4–3.7, 5.4
Alg. 5	Five stages perform exact modular doubling.	Formal result	Success-domain circuit action and a per-index reference-law bound are checked.	4.1
Alg. 6	Generic-prime doubling uses a truncated comparison and asserts a bounded high-part-gap event.	Conditional result	Success-domain action and reference-law bounds for the combined missed-reduction/discarded-carry event are checked.	4.3, 5.4
Alg. 7	Pseudo-Mersenne doubling asserts bounded missed-reduction and discarded-carry events.	Conditional result	Stage refinement, exact controlled value action, and cleanup are checked under the theorem’s input conditions.	4.1
Alg. 8	Five stages perform exact controlled modular addition.	Formal result	Success-domain action and a reference-law result for the combined arithmetic/cleanup event are checked.	4.4, 5.4
Alg. 9	Generic controlled addition asserts a bounded combined arithmetic/cleanup event.	Conditional result	Separate reference-law results retain their named prior-success conditions and dependence.	4.4, 5.4
Alg. 10	Pseudo-Mersenne controlled addition asserts bounded arithmetic and cleanup events.	Conditional result	Circuit action, three prioritized failure classes, and per-index and 402-index reference-law results are checked. The fixed checked multiplier sets <code>equalityIterations=50</code> .	4.5, 5.4
Alg. 11	Equality-aware controlled addition handles $x + y = p$.	Conditional result	Grouped baseline counts reproduce. Two widths and the full-window surcharge disagree. Total-gate aggregation and the paper-wide failure probability remain unsupported.	5, 6.2–6.5
Paper T1	Four point-addition rows have the printed widths, grouped costs, total-gate values, and failure bound.	Source reconstruction, source execution, and unresolved	The factor 28 and core arithmetic reproduce. Exact full-window-source arithmetic corrects two displayed exponents by 0.01. The Babbush check covers arithmetic over published bounds. Circuit-level verification remains open.	6.6–6.7
Paper T2	The point-addition rows compose to the displayed ECDLP exponents, including two Babbush comparisons.	Source reconstruction	The audit-inferred map matches 17 of 18 cells. Generic modular squaring is about 10.54 percent and rounds to 11. The authors’ class map and rounding order are absent.	6.8
Paper T3	The listed classes account for the printed percentage breakdown.	Source reconstruction and unresolved		

2 ECDLP reduction and point addition

2.1 Separately generated ECDLP reference circuit

The checked Section 2 endpoint is `ReferenceVerification` for a separately generated exact VQ ECDLP circuit. The circuit composes state preparation, a compiled exact two-fixed-base scalar-multiplication oracle, and two QFTs. Let r be the `secp256k1` subgroup order. At VQ phase level λ , amplitudes lie in a dyadic cyclotomic ring whose

distinguished root of unity has order 2^λ . The generated 256-qubit QFT uses phase gates through level 257, giving the theorem premise $\lambda \geq 257$. For a valid encoded point $Q = [d]P$ with $d < r$, the theorem identifies the emitted terminal state and proves normalization and the scalar-pair marginal. Set $N = 2^{256}$ and $j(t) = \lfloor (2Nt + r)/(2r) \rfloor$. The selected Fourier-peak pairs are $(j(k), j(dk \bmod r))$ for $1 \leq k < r$. Decoding yields $k \neq 0$ and $v = dk \bmod r$, after which the recovery procedure inverts k and validates the recovered public point. The selected pairs have total mass at least

$$\frac{r-1}{r} \frac{2401}{14641},$$

which is approximately 0.164, below one half. The Lean theorem names this internal order constant `q`. The report uses r to avoid conflict with the paper’s field-modulus notation. The proposition is `VQ.Tests.ECDLPFourierRecovery.secp256k1_reference_verification`. Its source and assumptions appear in Appendix A.

Schrottenloher’s implementation-level recovery claim requires a theorem connecting the Algorithm 1 point-addition oracle to this reference circuit. That theorem must relate the two-scalar oracle states and transfer the selected-pair recovery result to the paper execution. The retained evidence contains no such theorem. The checked selected-pair mass lower bound is about 0.164, so the greater-than-one-half claim remains unresolved.

The separately generated VQ reference oracle uses exact fixed-base scalar-multiplication circuits. Schrottenloher’s Algorithm 1 proposes a different windowed affine point-addition implementation. A paper-facing theorem must relate the two circuits at the oracle-state and resource levels.

2.2 Input-law obstruction

The paper starts from two uniform 256-bit basis-label registers, a finite source with $2^{256}2^{256}$ equally weighted labels. The reference law has $(p-1)p$ equally weighted pairs, because its nonzero Euclidean input has $p-1$ choices and its independent coefficient has p choices. If a deterministic function pushed the uniform basis-label law exactly to the uniform reference law, every target point would have a fiber of the same integer size. Hence $(p-1)p$ would divide 2^{512} .

The checked cardinality theorem rules out that divisibility. The odd prime p divides $(p-1)p$ but is coprime to every power of two, so it cannot divide 2^{512} . The formal statement `algorithm0BasisLabel_not_pushesForward_reference` therefore proves that no deterministic map from the two uniform 256-bit labels has the exact reference-law pushforward.

The exact pushforward identity is unavailable as a transfer theorem for the reference-law bounds. Alternative routes include an event-specific domination inequality, a coupling with controlled error, or a direct analysis of Algorithm 1 call states. The retained evidence supplies none of these, so Section 5 keeps every reference-law result conditional.

2.3 Algorithm 1: conditional affine-path semantics

The published generic affine path of Algorithm 1 has checked whole-register action and an affine-result formula under explicit premises. The `selector16_point_addition_act` theorem covers the generated sequence of lookups, modular subtraction, in-place division, controlled squaring, modular addition, negation, and final unlookup on its register layout. Let (a_x, a_y) be the selected point, (x, y) the target, and $e = \text{decide}(\text{address} \neq 0)$. With all arithmetic modulo p , set

$$\ell = \frac{y - a_y}{x - a_x}, \quad s = (x - a_x) + 3a_x, \quad g = \begin{cases} s - \ell^2, & e = \text{true}, \\ s, & e = \text{false}. \end{cases}$$

The `selector16_point_addition_affine` theorem proves that the target coordinates become

$$(x', y') = \begin{cases} (a_x - g, \ell g - a_y), & e = \text{true}, \\ (a_x + g, \ell g - a_y), & e = \text{false}, \end{cases}.$$

The source names this pair `affineResult`. The theorem also proves that the stated work regions return to zero.

The action and affine theorems require the point, lookup, subtraction, and division work regions to start at zero. The selected and target coordinates must be reduced, the selected shift must contain its required table value, the selected and target x coordinates must differ, and the adjusted division input must be positive. Separate lemmas cover the two control values. The theorem `selectedPoint_algorithm1_true` identifies the enabled affine result with `Curve.addPoint` when both input encodings represent secp256k1 group points, the condition named `GroupValid` in the formal source, their x coordinates differ, and the sum’s x coordinate differs from a_x . The source

calls the last condition the recovery-noncollision premise. The theorem `selectedPoint_algorithm1_false` proves target preservation when the selected coordinates are $(0,0)$, the target is representable, and its x coordinate is nonzero. The theorem `selectedPoint_groupAdd` equates `groupAddValue`, the packed result of `Curve.groupAdd`, with group addition when both input encodings represent group points. A total circuit-to-group result requires a composition of these value lemmas with the generated circuit.

The paper’s complete oracle still requires proofs of equal- x exceptional cases, the point at infinity, a complete circuit-to-group branch dispatcher, instantiated and validated point and shift tables, cleanup of persistent branch-selection state, a specified measured-unlookup circuit, a composite point-addition resource derivation, and the law of arithmetic arguments during superposed scalar multiplication. The whole-register formula and separate value lemmas apply under their displayed premises. A complete oracle theorem must discharge these obligations.

3 Euclidean trace, compressed record, and multiplier

3.1 Algorithm 2: transition and trace circuit

Algorithm 2 applies a binary-Euclidean transition to a pair of nonnegative remainders beginning at (p, x) . Each stored choice has an `odd` bit for the current right remainder and a `swap` bit for the ordered odd case. Validity requires that a set swap bit accompany a set odd bit. The transition halves an even right remainder or uses the ordered odd case to replace and halve the relevant difference. The terminal pair for a nonzero residue modulo the prime is $(1, 0)$, after which a fixed schedule can retain terminal padding choices.

The parameterized compressed-trace circuit has separate checked statements for its three observable parts. Starting from bounded input registers and clean carry, sign, comparison, and scratch wires, the input-pair theorem proves that the circuit registers contain the mathematical pair after the requested number of transitions. The record theorem proves that every five-bit block contains the corresponding compressed choices, and the cleanup theorem proves that the circuit clears the four temporary decision wires. These statements are parameterized by `rounds`. Section 3.5 gives a nonzero `secp256k1` input whose pair remains nonterminal after 402 rounds.

The publication shrinking schedule also has a conditional generated-circuit theorem. In `VQ/Experiments/SchrottenloherArtifactSchedule.lean`, `ArtifactTraceDomain` requires both full-width operands to fit the active field and every truncated comparison to agree with the full comparison at each scheduled round. The final `PaddingSafeAt` premise requires both operands to fit the final active field. The theorem `paperScheduleGates_action` also requires an odd initial left operand, a clear six-bit raw field, a clear four-bit work field, and zeros in the 236 initially inactive record positions. It proves the artifact-trace active pair, raw/work cleanup, and every compressed record block. `paperScheduleCircuit_wellFormed` checks circuit well-formedness. Total multiplier correctness additionally requires termination and every arithmetic-step success premise.

The trace and record claims also have different failure modes. A too-short schedule may record 402 correct transitions and still stop before the terminal pair. A shrinking register may select the correct comparison and then alter the next value by clipping it. Sections 3.5 and 3.7 give checked witnesses for these two distinct failures.

3.2 Figure 1: three-choice compression

One Euclidean choice is a pair of Boolean values subject to `swap` implying `odd`, leaving three valid choices. A block is an ordered triple of choices encoded on six wires, two wires per choice. The $3^3 = 27$ valid six-bit blocks fit in five bits. Valid-block readout means that, for any such block, the circuit’s six-bit output equals the compressed code and is below 2^5 , so its sixth wire is clear and its retained five bits determine the original triple.

The Figure 1 gate list has checked forward and inverse action on all 27 valid inputs. Copies placed on consecutive blocks have checked family readout and reverse to the original family input. Separately, reconstruction’s `decodeCompressedRecord` handles a final partial block containing one or two valid choices. Each full block uses exactly 13 gates: 5 CCX gates, 6 CNOT gates, and 2 single-bit NOT gates.

The paper’s 402 decisions occupy $402/3 = 134$ blocks and retain $134 \cdot 5 = 670$ bits. A deterministic 512-decision schedule occupies $\lceil 512/3 \rceil = 171$ blocks, including one partial block, and retains $171 \cdot 5 = 855$ bits. The two layouts therefore use the same checked compressor but have different record requirements.

3.3 Algorithm 3: reversed Bézout reconstruction

The pure reconstruction theorem follows an invariant between the current coefficient pair and the exact Euclidean remainder pair. Each reverse choice updates the coefficients by the algebraic inverse corresponding to its forward transition. Reading the compressed record backward reproduces the uncompressed choice sequence, including a

final block containing one or two valid choices. If the recorded trace from (p, x) is terminal and $y < p$, the checked pure result maps the coefficient pair to $(0, xy \bmod p)$ for odd p .

The pure result assumes a terminal trace and exact Euclidean arithmetic. The theorem `reconstructCompressedCompressedTraceCircuit_of_terminal` connects the record emitted by the compressor circuit to the mathematical reconstruction. The fixed $2n$ corollary derives terminality from primality, a positive nonzero residue, and the full-width $2n$ schedule.

The generated reverse-schedule theorem acts on finite-padding coefficient registers and assumes each named truncated or pseudo-Mersenne coefficient-step predicate. The schedule maps $(y, 0)$ to $(0, xy \bmod p)$ and cleans its temporary registers when those predicates hold for the recorded trace. The composite-multiplier theorem inherits the same success premises.

3.4 Algorithm 4: conditional multiplication

The in-place multiplier composes three reversible stages. The forward trace generates the remainder pair and compressed choices, the reverse reconstruction consumes those choices while transforming the multiplier target, and the inverse trace erases the Euclidean record and restores the input register. On a terminal trace satisfying the padding premise and the pseudo-Mersenne reconstruction success predicate, the checked 402-round pseudo-Mersenne circuit implements

$$(x, y) \mapsto (x, xy \bmod p)$$

and returns its record and work registers to their initial clean values.

The theorem accepts any $x < 2^{256}$ and $y < p$ for which the 402-step trace is terminal and the pseudo-Mersenne reconstruction success predicate holds. The convenience corollary packages those conditions as a reference-multiplier success predicate, which remains an input premise. Algorithm 1 affine composition, its input law, and the point-addition total in Schrottenloher’s Table 1 require separate theorems.

Under the reference law, the probability of a terminal input with a first approximate-reconstruction failure among the 402 reverse steps is below 2^{-29} . Termination failure and reconstruction out-of-domain are disjoint, and the multiplier out-of-domain probability equals their sum. That sum is below the 402-round termination-tail probability plus 2^{-29} . This fixed-model result retains the reference law and excludes the publication shrinking-field and truncated-comparison events.

3.5 Origin of the 402-round schedule

The paper models the binary-Euclidean iteration count with empirical mean $1.413n$, observed standard deviation about $0.6\sqrt{n}$, and schedule parameter $c_{\text{iter}} = 2.4$. The schedule source at revision `d8d2473cb43c` evaluates

$$3 \left\lceil \frac{1.413n + 2.4\sqrt{n}}{3} \right\rceil = 402$$

for $n = 256$, rounding to a multiple of three for the compressor. The cited binary-Euclid analysis labels the product-shrink and normal-tail steps used for this choice as heuristics [3]. Neither the paper nor that source supplies a fixed-denominator theorem for the secp256k1 residues whose traces exceed 402 rounds.

The 402-round tail must be bounded under the law induced by the complete algorithm. The paper does not derive the distribution of multiplier inputs inside Algorithm 1, and Section 2 rules out the proposed exact pushforward to the reference law. The million-residue audit concerns IID-uniform accepted nonzero residues. Its result does not bound Algorithm 1’s induced call law.

3.6 Checked counterexample and deterministic correction

One checked residue disproves universal termination of the 402-round pseudo-Mersenne specialization: it leaves the pair $(1, 7)$ after round 402 and reaches $(1, 0)$ at round 405. Appendix B gives the complete residue and theorem locations. Lean checks the witness directly.

For every live full-width transition, Lean separately proves the product inequality $2u'v' \leq uv$. The sharp termination proof uses strict descent of $\text{bitLength}(u) + \text{bitLength}(v)$ at every live step. Its trace inequality, with coprimality of (p, x) and the terminal bit-length sum, gives the universal $2n - 1 = 511$ round bound for nonzero secp256k1 residues. The input 2^{255} first terminates at round 511, showing that the 511-round upper bound is attained. A fixed 512-round reversible schedule with the exact adder therefore has a checked multiplication and cleanup theorem for every nonzero input and reduced target under clean-work and register premises.

The deterministic correction’s 171 compression blocks retain 855 record bits. The paper’s two shared 374-bit Euclidean fields provide 748 positions. The inequality $855 > 748$ is a checked incompatibility with that reported shared-field allocation. Corrected resource rows require a new layout and cost derivation.

3.7 Shrinking-field failure

The publication shrinking schedule has a value failure independent of the 402-round tail. For one checked residue, round 369 begins from (6033964351, 44029). The full comparison and the comparison on the retained bits choose the same branch. Clipping changes the successor from (44029, 3016960161) to (44029, 869476513) and loses high bits from a live operand before the next arithmetic update.

This witness concerns the publication-source shrinking model with 374-bit field allocation, 37 padding bits, comparison-truncation parameter 40, and effective checked comparison width equal to the smaller of the active field width and 77. The witness violates the **ArtifactTraceDomain** premise of the conditional schedule theorem. Under the uniform law on nonzero residues, the existence of the one displayed input gives the shrinking trace model’s value-failure probability the checked lower bound $1/(p-1)$. Appendix B records the complete input and divergent state data.

4 Exact and approximate modular operations

4.1 Algorithms 5 and 8: exact operations

Algorithm 5’s five stages implement exact modular doubling and clean their work registers for an odd modulus. The stages shift the widened target, load and subtract the modulus, conditionally add it back and unload the modulus, invert the subtraction flag, and cancel that flag with the low result bit. The stage lemmas refine the pseudocode values into one register theorem, including the equivalence between the paper’s cleanup order and the order in the selected reversible realization. Under $x < p$, $p \leq 2^n$, clean source and flag registers, and odd p , the final target is $2x \bmod p$ and all non-target registers return to their input values.

Algorithm 8’s five stages implement exact controlled modular addition and clean their work registers under its stated input conditions. Its refinement first conditionally adds the source to the target, subtracts p in a widened register, conditionally restores p , recomputes the comparison information needed to clear the borrow, and inverts the final ancilla. For reduced $x, y < p$, a clean workspace, and a Boolean control, the checked result is y when the control is false and $(x + y) \bmod p$ when it is true. The theorem also proves that the widened high bit and arithmetic work region are clear at the endpoint.

These results connect the pseudocode stage order to chosen reversible circuits, their register placement, and their compiled primitive gates. The isolated counts cover those implementations. Composite point-addition rows also require lookup, inversion, squaring, branch handling, and the source reporter’s cost convention.

4.2 Success domains and failure classifications

The checked approximate-operation theorems prove register action and cleanup under named arithmetic predicates. Separate classification lemmas locate selected predicate failures in intervals or arithmetic cases. Each conclusion applies to its named failure class and model. The associated probability theorems use the reference law and do not derive the distribution of Algorithm 1 call states.

Table 3: Circuit semantics and named events for approximate operations.

Alg.	Intended operation	Checked circuit result	Named failure classes	Probability result
6	Generic-prime modular doubling	Writes $2x \bmod p$ and cleans the selected workspace when the efficient-double success predicate holds.	Selected high-part-gap comparison event.	Per-index reference-law theorem. See Sec. 5.4.
7	Pseudo-Mersenne modular doubling	Writes $2x \bmod p$ and cleans the low-width workspace when the pseudo-Mersenne success predicate holds.	Missed reduction and discarded carry.	Per-index and 402-index-sum reference-law theorems. See Sec. 5.4.

Alg.	Intended operation	Checked circuit result	Named failure classes	Probability result
9	Generic-prime controlled modular addition	Writes the controlled sum and cleans its workspace on the efficient-add success domain.	Combined arithmetic and cleanup event for the generic construction.	Per-index reference-law theorem. See Sec. 5.4.
10	Pseudo-Mersenne controlled modular addition	Writes the controlled sum and cleans its workspace on the pseudo-Mersenne success domain.	Addition arithmetic after successful doubling. Cleanup after prior success.	Two conditional per-index reference-law theorems. See Sec. 5.4.
11	Equality-aware pseudo-Mersenne addition	Implements the equality-aware controlled sum and cleans its workspace when the equality-aware success predicate holds.	Detection, correction cleanup, and equality cleanup, in priority order.	Per-index control-split and 402-index-union theorems. See Sec. 5.4.

4.3 Algorithms 6 and 7: approximate doubling

Algorithm 6 uses only a retained high part to decide whether doubling crosses the modulus. The efficient-double circuit theorem states exact modular doubling and cleanup when the high-part comparison agrees with the reduction needed by the full value. A generic-prime probability module classifies a selected disagreement as a high-part-gap event. A checked secp256k1 theorem equates that event with the fixed secp256k1 efficient-prefix event. The circuit-action theorem remains separate. Transfer from the reference law to the point-addition oracle requires another theorem.

Algorithm 7 exploits the pseudo-Mersenne relation between p and 2^{256} . Its low-width computation can fail by missing a required modular reduction or by discarding a carry that affects the retained result. The circuit action is checked whenever neither event occurs, and separate classification lemmas describe the value-level failure intervals. The law and numerical bounds for their combined event are stated in Section 5.4.

Algorithm 6’s high-part gap belongs to the generic comparison model. Algorithm 7’s event uses the pseudo-Mersenne correction and carry structure. Assigning either bound to an Algorithm 1 division or addition call requires a theorem matching the call state and event.

4.4 Algorithms 9 and 10: approximate controlled addition

Algorithm 9 has checked success-domain action for generic-prime controlled addition. Its selected failure event combines an arithmetic discrepancy with the cleanup condition needed to restore the work register. The coefficient-fiber argument bounds that combined event under the reference law. Applying the Algorithm 9 bound to Algorithms 6 or 10 requires a theorem equating the event definitions.

Algorithm 10 has checked success-domain action for pseudo-Mersenne controlled addition. Its evidence separates the arithmetic event for an addition performed after a successful doubling from the cleanup event after all named prior operations succeed. The two bounds retain different conditioning statements. Composite use must preserve the circuit’s conditional order. The cleanup premises differ between the disabled and enabled control fibers. A checked $p = 13$ Algorithm 11 counterexample refutes a control-agnostic coefficient-fiber lemma.

4.5 Algorithm 11: equality-aware addition

Algorithm 11 adds equality handling to the pseudo-Mersenne controlled operation used in the first 50 forward rounds of the checked multiplier. Its circuit theorem implements the intended equality-aware sum and cleanup on the named success domain. The failure classification orders three events: detection fails to recognize the exact equality condition, correction cleanup leaves information after the equality correction, or final equality cleanup leaves a low-block residue. Priority prevents one input from being charged under a later class after an earlier class has already occurred.

The final probability proof splits on the control. The disabled branch and enabled branch have different coefficient-fiber structures, so each receives the event definition supported by its algebraic premises before the results are recombined. This split supplies the checked per-index and 402-index reference-law bounds.

5 Failure-probability analysis

5.1 Complete failure event and call-law transfer

Schrottenloher’s Table 1 assigns one probability to complete circuit failure. The audit identifies five failure families required by that aggregate claim: failure to terminate within 402 rounds, invalidity introduced by shrinking live fields, a wrong branch from truncated comparison, approximate modular arithmetic failure, and incomplete cleanup. The 405-round witness concerns termination. The round-369 witness concerns shrinking-field invalidity following a correct comparison. The Algorithm 4 reference-law theorem combines termination and every approximate-reconstruction event in the fixed pseudo-Mersenne model. It does not include the publication shrinking-field and truncated-comparison events or transfer to Algorithm 1 call states. The checked record contains no definition or bound for the union of all five families in the paper circuit.

Every arithmetic-event bound for Algorithms 4, 6, 7, and 9–11 uses the reference law defined in Section 1. Algorithm 1 produces a sequence of arithmetic calls from the paper’s two-register basis-label law. The audit has no theorem deriving the reference law for those calls and no event-specific domination theorem for their induced law. Calls and conditional events may be dependent.

The paper-wide event must contain all five failure families in one probability space. Its conditional decomposition must follow the circuit’s execution order. A transfer theorem must relate each local reference-law event to the corresponding call-state event.

5.2 Zero failures in 10,000 tests

The paper reports 10,000 random tests with no observed failures and prints $2^{-13.3}$ in Schrottenloher’s Table 1 caption. The schedule source at revision `d8d2473cb43c` contains the simulation procedure and uses deterministic worker seeds. The publication bundle omits the sampled inputs and output transcript. A binomial interpretation adds an IID Bernoulli premise that the retained source does not establish. The reported zero count describes 10,000 sampled executions of the generated circuit.

The million-residue protocol selects $1/10100$ as a rational null boundary. Lean checks $1/10100 < 2^{-13.3}$ through the integer inequality $2^{133} < 10100^{10}$. The rational null boundary therefore lies below the captioned rate.

Suppose IID trials had true failure probability $1/10000$. The probability of seeing zero failures would be

$$\left(\frac{9999}{10000}\right)^{10000},$$

which the audit bounds between one third and one half. The rate $1/10000$ exceeds both $1/10100$ and $2^{-13.3}$. Under that rate, the reported zero count still occurs with probability in that interval.

The one-sided 95-percent Clopper–Pearson upper endpoint after zero failures in 10,000 trials is

$$1 - 0.05^{1/10000} \approx 3.00 \times 10^{-4}$$

under the binomial model [4]. Both $1/10100$ and $2^{-13.3}$ lie below this endpoint. The 95-percent interval therefore includes rates above the captioned upper bound. The sampled executions supply no universal result for the generated circuit.

5.3 Million-residue termination audit

The external termination audit samples 32-byte big-endian candidates and accepts a candidate exactly when its integer value x satisfies $1 \leq x < p$. Under the stated premise, accepted nonzero residues are IID and uniform on $\{1, \dots, p-1\}$. Its failure classifier is $L_p(x) > 402$, where $L_p(x)$ is the full-width binary-Euclidean trace length from (p, x) .

The tracked manifest and result document record 1,000,000 accepted candidates, no rejections, 19 candidates classified as longer than 402 rounds, and a maximum length of 415. Conditional on the IID-uniform model and recorded provenance, the one-sided 95-percent Clopper–Pearson endpoint is about 2.79×10^{-5} , below $1/10100$. The observed fraction is $19/1,000,000$. The endpoint accounts for sampling uncertainty in that fraction.

Lean checks the trace lengths of the 19 listed failures and 18 checkpoints, exact binomial definitions, a rational test case, and the integer comparison placing $1/10100$ below $2^{-13.3}$. A dependency-free evaluator encloses the million-trial binomial calculation. A transcript checker compares the listed positions and values with the raw bytes. The IID premise, transcript provenance, composite-null monotonicity, million-trial numerical enclosure, and Clopper–Pearson bracket remain external to Lean. The 32 MB raw transcript has no durable URI. Appendix D reports the tracked record and its retrieval limit.

5.4 Conditional arithmetic-event bounds

Every numerical result in Table 4 fixes the secp256k1 modulus and uses the reference law. An index denotes the selected reverse Euclidean step in the checked 402-round model. The aggregate rows use a union bound or a sum of per-index bounds, both valid under arbitrary dependence.

Table 4: Checked arithmetic-event probability bounds under the reference law.

Alg.	Event	Range	Checked numerical result and retained condition
4	Terminal input with approximate reconstruction outside its checked domain	First failure across 402 reverse indices	Below 2^{-29} in the fixed pseudo-Mersenne model.
4	Multiplier outside its checked domain	Termination or approximate-reconstruction failure	Below $\Pr[L_p(X) > 402] + 2^{-29}$. The termination tail remains a separate term.
6	Selected generic high-part-gap event	One reverse index	At most $(2^{216} - 1)/p$. A checked equivalence identifies the generic and fixed secp256k1 efficient-prefix events.
7	Pseudo-Mersenne missed reduction or discarded carry	One reverse index and sum over 402	Below 2^{-40} per index and below 2^{-31} for the sum over 402 indices.
9	Generic controlled-add arithmetic or cleanup event	One reverse index	Below 2^{-39} for the combined event.
10	Addition arithmetic after successful doubling	One reverse index	Below 2^{-40} under the named successful-doubling condition.
10	Cleanup after prior success	One reverse index	Below 2^{-40} under the named prior-success condition.
11	Prioritized detection, correction-cleanup, or equality-cleanup event after control split	One reverse index and union over 402	Below 2^{-38} per index and below 2^{-29} for the 402-index union.

Rows 6–11 give component-event bounds. The Algorithm 4 reconstruction row combines prioritized first failures of Algorithms 7, 10, and 11 across all 402 reverse indices on terminal inputs. Termination and reconstruction failure are disjoint, so the multiplier row adds their probabilities. These Algorithm 4 rows exclude the publication shrinking-field and truncated-comparison events and retain the reference law. The failure probabilities for Algorithm 1 and the complete ECDLP circuit remain unresolved.

5.5 Implications required for Schrottenloher’s Table 1 failure bound

A proof of Schrottenloher’s Table 1 aggregate failure probability requires four implications. For each Algorithm 1 call, either the reference law must be derived or the selected event must be dominated under the induced call law. The 402-round termination tail must be bounded under that law, or a replacement schedule must supply universal termination. Termination, shrinking-field, comparison, arithmetic, and cleanup failures must be events in one circuit model. Their union must be bounded in the computation’s conditional order.

The deterministic 512-round theorem supplies universal termination for a corrected schedule. Its 855-bit record exceeds the paper’s 748-position shared-field layout. A revised construction requires a larger record layout, the Algorithm 1 input law, the remaining event implications, and new probability and resource rows. The published construction leaves these four implications open.

6 Resource-source audit

6.1 Source versions and execution method

The resource executions use publication companion revision `3b60d5050b18` [8]. They use Qarton revision `7506faba2147` [7]. The four schema-v2 capture suites ran in one uv-managed CPython 3.12.12 environment with Qarton 0.2.1 and the package versions listed in Appendix D. Two suites used the companion’s baseline table $(0G, G)$, and two directly instantiated the pinned point-addition class with all 65,536 rows. Every mode exited with status zero and empty standard error. Corresponding results are byte-identical across the two baseline suites and across the two full-window suites.

The schema-v2 manifests and canonical result JSON files preserve source identities, package versions, commands, output hashes, and clean pre/post source states for the August 2026 execution. The authors’ June 2026 environment

remains unidentified because the companion gives dependency lower bounds and uses a mutable base image. Exact reconstruction requires a dependency lock and an immutable base-image identifier.

Static source arithmetic evaluates formulas and constants transcribed from the pinned Python source. Captured reporter output records what that Python program emitted under the retained environment. Lean checks equality between the static source formulas and the captured integers. The audit contains no syntax-to-event or syntax-to-liveness theorem for the Python program.

6.2 Schrottenloher’s Table 1 baseline counts

For these captures, the grouped metric is raw CCX plus raw CCZ plus charged forward temporary AND. Every retained result reports zero raw CCZ, so the grouped count equals raw CCX plus forward AND. Measurement-based AND uncomputation appears as a separate reporter count and receives zero grouped cost under the pinned convention. Other charged gates in the three measured-unlookup circuits contribute 4,584 grouped units to the full-window surcharge.

Mode	Captured grouped	Captured width	Paper width
Special-prime, space	2,385,517	1,192	1,192
Special-prime, gate	1,865,521	1,445	1,446
Generic-prime, space	3,608,043	1,191	1,192
Generic-prime, gate	2,803,519	1,446	1,446

Table 5: Baseline grouped counts and peak widths in the paper’s row order.

Static source reconstruction and captured execution agree on all four grouped counts in Table 5. The captures report widths 1,192, 1,445, 1,191, and 1,446 in paper row order. The paper reports 1,192, 1,446, 1,192, and 1,446. The special-prime gate and generic-prime space rows therefore differ by one qubit. Appendix C separates raw CCX, forward AND, measured uncompute, grouped total, and basic reporter sum for every mode.

The exact grouped counts have nearest-hundredth base-two exponents 21.19, 20.83, 21.78, and 21.42 in the paper’s row order. These values reproduce the four displayed baseline non-Clifford entries under the pinned grouped-cost convention. The total-gate and failure-probability columns remain unresolved.

6.3 Peak-width evidence

The captured root events defined in Section 1 reproduce the four baseline widths through Equation (1). Each baseline event has old peak 1,031, old and new current 514, and 512 previous qubits inside the mapped multiplier. The four child peaks are 1,190, 1,443, 1,189, and 1,444 in paper row order. The third recurrence term includes two live root qubits and gives widths 1,192, 1,445, 1,191, and 1,446.

The full-window root events have old peak 1,057, old and new current 529, and the same 512 previous-inside qubits and child peaks. Their recurrence term includes 17 live root qubits and produces 1,207, 1,460, 1,206, and 1,461. These five-tuples are retained instrumentation records at Qarton revision 7506faba2147, whose source locator for the recurrence appears in [7]. Arithmetic over each event confirms the recorded width. The checked evidence contains no theorem deriving an event from the Python program.

Pinned Qarton contains an asymmetric SWAP state-tracking transition in the mapping case that assigns operand zero as quantum and operand one as classical. The instrumentation labels this case `qc`. The schema-v2 records report zero reached `qc` SWAP events, zero faulty-branch events, and zero incorrect transitions in all retained runs. Every retained run avoids the affected branch. Both baseline width discrepancies occur in those runs [6].

6.4 Full-window construction and surcharge

The publication command builds the point-addition circuit with a two-row lookup table. The resource discussion assumes a 2^{16} -row window. The audit’s full-window runner invokes the pinned `WindowAffAddSpaceOpt` class with rows $([0]G, [1]G, \dots, [65535]G)$ and the paper’s four choices of prime specialization and gate/space mode. The full-window records capture these executions of the point-addition class.

The source-derived surcharge has three parts. Three forward lookups cost 196,602 grouped units. Charged select-swap and forward-AND gates inside three measured-unlookup circuits add 4,584, with zero cost assigned to the measured-AND-uncompute events. Four nonzero tests add 224. The sum is

$$196,602 + 4,584 + 224 = 201,410,$$

which exceeds the paper’s 196,608-unit surcharge by 4,802. Replacing the baseline one-bit address with a 16-bit address raises the reported peak width by 15 qubits. The paper adds 16 qubits.

Mode	Full-window grouped	Full-window width	Baseline increment
Special-prime, space	2,586,927	1,207	201,410 / 15
Special-prime, gate	2,066,931	1,460	201,410 / 15
Generic-prime, space	3,809,453	1,206	201,410 / 15
Generic-prime, gate	3,004,929	1,461	201,410 / 15

Table 6: Full-window captures and grouped-count/width increments.

All four captured grouped increments equal the source reconstruction. The measured width increments agree with the address replacement and captured root events. A whole-ECDLP resource theorem also requires the complete two-scalar schedule, initial lookup, Fourier transforms, measurement procedure, and their composition. The captures instantiate one point-addition class.

6.5 Reporter sums and Schrottenloher’s Table 1 total-gate column

The schema-v2 baseline result JSON files report `basic_gate_counts()` dictionary sums of 19,929,384, 31,170,396, 35,852,486, and 37,862,666. The full-window files report 66,002,343, 77,243,355, 81,925,445, and 83,935,625. Each value sums every dictionary entry returned by the reporter. The publication source gives no operation set, aggregation rule, or rounding rule for Schrottenloher’s Table 1 total-gate column. That column remains unresolved.

6.6 Schrottenloher’s Table 2 schedule and arithmetic

Two 256-bit scalar registers contain $2 \cdot 256/16 = 32$ window ordinals. The schedule replaces the first point addition by a direct lookup and defers the final three additions, leaving $32 - 1 - 3 = 28$ retained point additions in the displayed core formula. The formula contains no term for the initial direct lookup. The paper gives no numerical bound for that lookup.

Table 7 recomputes the paper using its rounded point-addition inputs and stated 196,608 surcharge, then gives exact baseline counts with that surcharge and exact full-window source counts multiplied by 28. The full-window captures supply the corrected widths directly. They differ from the widths obtained by adding the paper’s stated 16 qubits.

Mode	Paper rounded-input $\log_2$	Exact baseline + paper surcharge $\log_2$	Exact full-source $\log_2$	Paper width	Full width
Special-prime, space	26.11	26.11	26.11	1,208	1,207
Special-prime, gate	25.78	25.78	25.79	1,462	1,460
Generic-prime, space	26.66	26.67	26.67	1,208	1,206
Generic-prime, gate	26.33	26.32	26.33	1,462	1,461

Table 7: Three calculations for Schrottenloher’s Table 2 in its row order.

The exact full-source exponents are 26.11, 25.79, 26.67, and 26.33. The second and third values correct the corresponding printed 25.78 and 26.66 by 0.01. The exact full-source totals are 72,433,956, 57,874,068, 106,664,684, and 84,138,012. Appendix C gives the parallel baseline-plus-paper-surcharge totals and every substitution. These totals evaluate the 28-addition core formula. The audit contains no source-derived resource result for the complete ECDLP circuit.

6.7 Babbush comparison rows

Babbush et al. report average executed CCX-plus-CCZ point-addition bounds of 2.7 million in the low-qubit construction and 2.1 million in the low-gate construction, with respective point-addition qubit bounds 1,175 and 1,425 [2]. Their reported average point-addition bounds derive from 9,024 hash-selected test cases. Schrottenloher’s Table 1 displays those bounds after rounding as point-addition exponents 21.36 and 21.00. Equations (A1)–(A3) in Babbush et al. separately compose the published point-addition quantities into their core estimates.

Substitution into Schrottenloher’s 28-addition comparison gives

$$\begin{aligned} 28(2,700,000 + 3 \cdot 2^{16}) &= 81,105,024, & 1175 + 16 &= 1191, \\ 28(2,100,000 + 3 \cdot 2^{16}) &= 64,305,024, & 1425 + 16 &= 1441. \end{aligned}$$

Their base-two gate exponents are 26.27 and 25.94, reproducing Schrottenloher’s displayed comparison rows. The formal evidence checks these integer substitutions and the associated rounding.

The circuits used for the Babbush et al. estimates are withheld. Their public Zenodo v2 deposit has DOI 10.5281/zenodo.19597130. It contains the program, public outputs, verification keys, and Groth16 proofs [1]. The audit did not verify the Zenodo bundle. The published equations give no numerical upper bound for the initial direct lookup, lookup uncomputation, or recycled-QFT work. The reconstructed values cover the 28-addition core arithmetic over published point-addition bounds. Exact totals for a released whole circuit remain unavailable.

6.8 Schrottenloher’s Table 3 class reconstruction

Qarton’s recursive class reporter records costs at both parent and child classes and uses separate names for forward and inverse classes. The paper provides no class-selection map, overlap policy, or rounding order for Schrottenloher’s Table 3. The audit infers a rule that selects named captured classes, applies Qarton’s `round(numerator/denominator, 2)` to each selected recursive class, and then sums the corresponding integer percentages. The authors’ rule remains unavailable.

The inferred rule matches 17 of 18 printed cells. In the generic-prime space capture, modular squaring has numerator 380,314 and grouped denominator 3,608,043. Their ratio is about 10.54 percent. The inferred rule rounds this value to 11 percent. The table prints 10 percent. Classes that individually round to zero make reverse inference nonunique even when a row total agrees.

The special-prime controlled-modular-adder capture admits two class selections. Selecting the reported controlled modular adder and its inverse gives the printed 39 percent. Adding the separately captured pair of `HandleP` classes contributes another six percentage points and gives 45 percent. The source provides no rule for excluding those classes. The complete inferred class map and both denominators appear in Appendix C.

7 Claim boundaries between formal checks and captured executions

The formal record is VQ revision 00ce72179927, with theorem sources under `VQ/Experiments`, paper-facing tests under `Tests`, and the exact theorem index in Appendix A. The retained audit report is `paper/schrottenloher-final-audit.md`. The aggregate job is `schrottenloher-final-synthesis-aggregate-20260821-1`. The aggregate command `lake --rehash --reconfigure build Tests` completed 4,596 of 4,596 build jobs with status zero under Lean 4.32.0. The status-zero build establishes that the named statements and their imports checked at the pinned revision. Each theorem retains its stated premises and circuit domain. External-byte provenance and composition require separate evidence.

The source-execution record consists of the directories `schema-v2-baseline` and `schema-v2-full-window` under `paper/schrottenloher-resource-capture`. Both directories retain two manifests and four result files under the same names. Four tracked programs created and tested the records.

Record files:

```
manifest-run-1.json
manifest-run-2.json
special-prime-space-efficient.result.json
special-prime-gate-efficient.result.json
generic-prime-space-efficient.result.json
generic-prime-gate-efficient.result.json
```

Programs:

```
tools/schrottenloher-resource-capture.py
tools/test-schrottenloher-resource-capture.py
tools/run-schrottenloher-resource-captures.py
tools/test-run-schrottenloher-resource-captures.py
```

The static source record is `paper/schrottenloher-resource-audit.md`, supplemented by the checked arithmetic modules for Schrottenloher’s Tables 1–3. The 402-round schedule-source and probability-evidence audit is `paper/schrottenloher-402-source-audit.md`. The round-369 value counterexample is in the formal modules and tests indexed in Appendix A. Appendix D identifies retained capture programs, runner, tests, manifests, and payloads by SHA-256 and records manifest hashes for the inspected source trees.

The statistical record comprises exactly the five tracked files named in Section 1. Lean theorems check the listed trace lengths, checkpoints transcribed from the manifest, binomial definitions, a rational test case, and the rational threshold comparison. The million-trial sample, its raw-byte provenance, and the decimal interval enclosures come from external execution under the manifest’s stated environment. The missing raw transcript URI prevents independent retrieval of the exact 32 MB sample from the evidence repository.

The bibliographic record is Schrottenloher v1, the pinned companion and Qarton revisions, the schedule source, the cited binary-Euclid analysis, Babbush et al. v2 and DOI-tagged public bundle, and the Clopper–Pearson paper. The Babbush comparison checks Appendix arithmetic and rounding from their published v2 bounds. The audit record contains no check of the Zenodo program, outputs, verification keys, or Groth16 proofs.

Lean 4.32.0 checked the formal declarations and their proof terms [5]. The declarations import Mathlib [14]. Their paper-facing interpretation depends on faithful transcription of the paper’s parameters and source quantities. The captured results depend on CPython, Qarton, the installed packages, and the instrumentation that records root events and SWAP status. The statistical conclusions depend on the sampler, IID premise, transcript checker, and external interval evaluator. A publication lockfile and immutable image identifier are required to reproduce the authors’ software environment. A durable raw-transcript record is required to retrieve this audit’s sample bytes.

8 Evidence required to close unresolved claims

Table 8: Evidence that would change an unresolved disposition.

Unresolved item	Missing evidence	Conclusion supported by that evidence
The 402-round schedule has the stated tail	A fixed-denominator secp256k1 theorem under the induced multiplier-input law, or a revised deterministic schedule with its final layout and point-addition cost	A valid termination contribution to the paper’s aggregate bound, or a costed replacement for that contribution
Algorithm 1 supplies the reference-law arithmetic inputs	A theorem deriving its arithmetic-call law from the basis-label execution, or event-specific domination bounds for every selected call	Transfer of the conditional component bounds to the generated point-addition computation
Schrottenloher’s Table 1 has the stated complete failure probability	One circuit event space containing termination, shrinking-field validity, truncated comparisons, modular arithmetic, and cleanup, with a union theorem respecting every conditional premise	A checked aggregate bound for the exact paper circuit and input law
Schrottenloher’s Table 1 total-gate estimates follow from source	The included gate types and weights, summation or aggregation rule, and rounding rule	Reproduction or correction of the printed total-gate column from pinned reporter data
Schrottenloher’s Table 3 percentages follow from source	The authors’ class-selection map, inverse-class treatment, overlap policy, and per-class or post-sum rounding order	A unique reconstruction of all 18 cells and resolution of the generic modular-square discrepancy
The resource execution reproduces the publication environment	An exact dependency lock, immutable base-image identifier, and publication command that constructs the intended full lookup table	Re-execution under the authors’ recorded software environment
Captured widths follow from program syntax	A derivation from Python construction and memory-manager transitions to every dominant root event and peak-liveness value	A syntax-derived width result independent of instrumentation provenance
Schrottenloher’s Table 2 is a whole-circuit result	Generated whole-circuit syntax or separate bounds for the initial lookup, QFTs, measurement, and their composition and liveness	A whole-circuit interpretation with all named terms accounted for
Babbush circuit-level comparison	Released circuits with a checkable semantic and resource model, plus bounds for omitted lookup and recycled-QFT terms	Circuit-level checking and an end-to-end whole-circuit resource interpretation
Babbush attested computation	An independent audit of the public program, outputs, verification keys, and Groth16 proofs	Validation of the attested resource-and-test computation

Changing a disposition requires evidence for the same circuit model, input law, register layout, and resource convention. A component theorem under another model requires a checked transfer theorem. That theorem must establish the corresponding paper-facing claim.

9 Verified claims, corrections, and remaining gaps

The checked functional components include the separately generated exact ECDLP reference circuit and its selected-pair recovery result, Algorithm 1’s generated register path and affine-result formula under explicit premises, the

binary-Euclidean trace and compressed record, pure reversed Bézout reconstruction, the conditional in-place multiplier, and the exact modular operations in Algorithms 5 and 8. Separate Algorithm 1 value lemmas cover enabled addition and disabled zero selection under their respective premises. A circuit-to-group theorem for Algorithm 1 and a theorem relating that oracle to the ECDLP reference circuit remain open. Algorithms 6, 7, and 9–11 have checked circuit action on their named success domains and reference-law probability results for their selected events. Under the reference law, the probability of a terminal input with an approximate-reconstruction failure at one of the 402 reverse indices is below 2^{-29} , and multiplier domain failure is below the termination tail plus 2^{-29} . That aggregate covers the fixed pseudo-Mersenne model and retains the reference-law premise.

The Euclidean findings include a checked residue that needs 405 rounds and a deterministic alternative based on the tight universal full-width bound 511. The resulting 512-round exact-adder schedule needs 855 record bits, exceeding the paper’s 748-position shared allocation by 107. A separate round-369 witness has an agreeing comparison followed by divergent clipped and full successors, violating the publication shrinking schedule’s trace-domain premise. Resource executions reproduce the four grouped baseline counts and identify discrepancies in two baseline widths. Static arithmetic over the full-window source establishes the 201,410 surcharge. It gives Table 2 exponents 25.79 and 26.67 where the paper prints 25.78 and 26.66. Captured output and Qarton’s recurrence establish the 15-qubit peak-width increment. Under the audit-inferred map for Schrottenloher’s Table 3 and rounding order, generic modular squaring is about 10.54 percent and rounds to 11 percent. The paper prints 10 percent, and the authors’ map is absent.

The aggregate failure probability in Schrottenloher’s Table 1 and the whole elliptic-curve discrete-logarithm resource claims remain unresolved. Closing them requires the induced Algorithm 1 call law or event-specific domination, a common event space and conditional union theorem, a supported termination schedule and layout, the missing resource aggregation rules, and stronger provenance for the publication and Babbush computations. Those additions would determine whether the checked component results compose to the paper’s complete circuit.

A Formal evidence index

Every declaration and test in Table 9 belongs to VQ revision 00ce72179927. Each row summarizes the proposition and selected premises. The source declaration supplies the full type. The test module identifies the paper-facing check.

Table 9: Declaration, proposition, premises, and test index.

Item	Exact declaration and source	Proposition checked	Selected premises	Test module
§2	ReferenceVerification; secp256k1_reference_verification Tests/ECDLPFourierRecovery/Verified.lean	Emitted state of the separately generated exact VQ ECDLP reference circuit, normalized marginal, mass of selected Fourier-peak pairs $(j(k), j(dk \bmod r))$, selected-pair recovery, and circuit resources	VQ phase level $\lambda \geq 257$; valid point; $d < r$; $Q = [d]P$	Same file
§2 reference law	referenceMultiplierLaw_mass; referenceMultiplierLaw_probability VQ/Experiments/SchrottenloherFiniteProbability.lean	Exact point mass and event-cardinality formula for the fixed reference law	Uniform nonzero Euclidean input and independent uniform reduced coefficient	Tests/SchrottenloherFiniteProbability.lean
§2 input law	card_algorithm0BasisLabel; algorithm0BasisLabel_not_pushesForward_reference VQ/Experiments/SchrottenloherPaperInputBridge.lean	The two paper basis-label registers contain 2^{512} labels. No deterministic map pushes their uniform law exactly to the reference law.	Arbitrary candidate map and finite uniform laws	Tests/SchrottenloherPaperInputBridge.lean
Alg. 1	selector16_point_addition_wellFormed; selector16_point_addition_act; selector16_point_addition_affine; selectedPoint_groupAdd; selectedPoint_algorithm1_true; selectedPoint_algorithm1_false Tests/SchrottenloherAlgorithm1.lean; algorithm1Pre_iff VQ/Experiments/SchrottenloherAlgorithm1Assumptions.lean	The generated path is well formed and yields the stated whole-register affine result. Separate lemmas identify the enabled result with Curve.addPoint, the disabled $(0, 0)$ result with the target, and groupAddValue with group addition.	Clean work, reduced values, valid shift, unequal x , and positivity for the circuit. GroupValid, unequal x , and recovery noncollision for enabled addition. Representable target and nonzero target x for disabled zero selection	Tests/SchrottenloherAlgorithm1.lean
Alg. 2	compressedTraceCircuit_inputPair; compressedTraceCircuit_record; compressedTraceCircuit_clean VQ/Experiments/SchrottenloherCompressedTraceSemantics.lean	Parameterized circuit produces the mathematical pair and compressed record and clears decision scratch	Positive width; odd left input; both inputs fit the width	Tests/EuclidInPlaceMul.lean
Alg. 2 source	paperScheduleLegal; paperScheduleCircuit_wellFormed; paperScheduleGates_action VQ/Experiments/SchrottenloherArtifactSchedule.lean	Publication shrinking schedule has conditional circuit action, raw/work cleanup, and record-block semantics	ArtifactTraceDomain; odd initial left operand; final PaddingSafeAt; clear six-bit raw field; clear four-bit work field; zeros in the 236 initially inactive record positions	Tests/SchrottenloherArtifactSchedule.lean
Fig. 1	validChoice_card; validChoiceTriple_fits_five_bits; compressedChoiceTriple_lt; compressedChoiceTriple_injective; paperCompressionGates_length; paperCompressionGates_ccx; paperCompressionGates_cx; paperCompression_last_bit_zero; paperCompression_reverse VQ/Experiments/EuclidInPlaceMul.lean; paperCompressionFamily_read_valid_block; paperCompressionFamily_valid_block_lt; paperCompressionFamily_reverse VQ/Experiments/SchrottenloherCompression.lean	Three valid choices fit five bits. The compression map is bounded and injective. The 13-gate compressor has the checked CCX/CNOT counts, clears the sixth bit, reverses valid triples, and supports valid placed-block readout.	Valid choice triple and in-range family block	Tests/EuclidInPlaceMul.lean

Item	Exact declaration and source	Proposition checked	Selected premises	Test module
Alg. 3	<code>decodeCompressedRecord_compressedTraceRecord; reconstructCompressed_of_terminal; reconstructCompressed_compressedTraceCircuit_of_terminal; reconstructCompressed_two_mul_bits_of_prime; reconstructCompressed_compressedTraceCircuit_two_mul_bits_of_prime VQ/Experiments/SchrottenloherCompressedReconstruction.lean</code>	The decoder recovers the choices emitted by the generated trace circuit. Mathematical reconstruction from that record yields $(0, xy \bmod p)$ after a terminal trace.	Odd modulus; operands fit the width; terminal trace. The fixed- $2n$ prime corollaries require $0 < x < p$ and $y < p$.	Tests/EuclidInPlaceMul.lean
Alg. 3 generated	<code>circuit_act; circuit_traceInput_act; circuit_two_mul_bits_of_prime VQ/Experiments/SchrottenloherPseudoMersenneRawChoiceSchedule.lean</code>	Generated reverse schedule has the selected coefficient-step action and prime-width multiplication corollary	Odd prime and width fit; nonzero reduced multiplier input; reduced coefficient; low-width and kept-bit bounds; exact-adder interface; initialized fields; clear work fields; per-step success premises	Tests/SchrottenloherPseudoMersenneRawChoiceSchedule.lean
Alg. 4	<code>circuit_multiplies; circuit_multiplies_of_referenceMultiplierSuccessConditions VQ/Experiments/SchrottenloherPseudoMersenneSecp256k1.lean</code>	Checked 402-round circuit maps (x, y) to $(x, xy \bmod p)$ and restores its trace state	Input bounds; terminal 402-step trace; reconstruction and approximate-success premises	Tests/SchrottenloherPseudoMersenneSecp256k1.lean
Alg. 4 corrected	<code>correctedCircuit_act VQ/Experiments/SchrottenloherDeterministicCorrection.lean</code>	Exact-adder 512-round circuit multiplies and cleans for the corrected layout	secp256k1 primality; nonzero reduced source and reduced target; exact modular-adder interface; initialized source and target fields; zero work field	Tests/SchrottenloherDeterministicCorrection.lean
Alg. 4 reference-law aggregate	<code>referenceMultiplierReconstructionOutOfDomainProbability_lt_two_pow_neg_29; referenceMultiplierOutOfDomainProbability_lt_shortSchedule_add_two_pow_neg_29 VQ/Experiments/SchrottenloherPseudoMersenneFailureProbabilityRational.lean</code>	Under the reference law, the probability of terminal inputs with a first reconstruction failure among the 402 reverse indices is below 2^{-29} . Multiplier domain failure is below the short-schedule tail plus 2^{-29} .	Fixed secp256k1 402-round pseudo-Mersenne model; equality-aware addition during the first 50 forward rounds; reference law; secp256k1 primality	Tests/SchrottenloherPseudoMersenneSecp256k1.lean
Alg. 4 termination	<code>euclidStep_product_double_le; euclidTrace_product_bound; euclidStep_bitLengthSum_lt; euclidTrace_steps_add_final_bitLengthSum_le; traceSucceeds_two_mul_bits_sub_one_of_coprime; traceSucceeds_two_mul_bits_sub_one_of_prime VQ/Experiments/EuclidInPlaceMul.lean; all_nonzero_inputs_succeed_511; termination_bound_is_tight Tests/SchrottenloherDeterministicCorrection.lean</code>	Product shrink is a separate invariant. Strict bit-length-sum descent gives the 511-round universal bound, and 2^{255} proves tightness	Positive bit width; odd modulus; both operands below 2^{bits} ; coprimality. The prime specialization requires $0 < x < p$.	Tests/SchrottenloherDeterministicCorrection.lean
Alg. 4 shrinking	<code>domainFailureState_step_mismatch_369 Tests/SchrottenloherArtifactTrace.lean; one_div_p_sub_one_le_valueFailureProbability VQ/Experiments/SchrottenloherArtifactTraceFailure.lean</code>	One publication-schedule input has divergent full and clipped successors. Its point mass lower-bounds uniform-nonzero failure	Named witness; publication shrinking model; uniform nonzero law for probability	Tests/SchrottenloherArtifactTrace.lean; Tests/SchrottenloherArtifactTraceFailure.lean
Alg. 5	<code>paperLocalGates_act; circuit_refines_algorithm5; circuit_doublesMod; secp256k1_program_width; secp256k1_program_toffoliCount; secp256k1_program_cnotCount VQ/Experiments/SchrottenloherAlgorithm5.lean</code>	Five-stage circuit refines the pseudocode and doubles modulo an odd modulus. The isolated secp256k1 program has width 517, 3,340 Toffoli gates, and 1,542 CNOT gates.	Odd modulus, fitting width, reduced input, and clean work	Tests/SchrottenloherAlgorithm5.lean
Alg. 6	<code>circuit_act VQ/Experiments/SchrottenloherEfficientDoubleBoundary.lean; algorithm6FailureProbability_le VQ/Experiments/SchrottenloherGenericPrimeAlgorithm6Probability.lean; algorithm6Secp256k1FailureBridge_event_eq VQ/Experiments/SchrottenloherGenericPrimeAlgorithm6Secp256k1Bridge.lean</code>	Success-domain doubling action; generic high-part-gap bound; equality of generic and fixed secp256k1 efficient-prefix events	Circuit action: odd modulus and modulus fit; reduced initialized operand; zero workspace; kept width at most the operand width; efficient-double success. Probability: prime parameters and an Algorithm 6 failure bridge with the block bound, fiberwise injective gap encoding, and failure classification. The fixed secp256k1 event requires a terminal trace and successful efficient prefix before double failure, under the uniform reference law.	Tests/SchrottenloherEfficientDouble.lean; Tests/SchrottenloherGenericPrimeAlgorithm6Secp256k1Bridge.lean
Alg. 7	<code>circuit_act VQ/Experiments/SchrottenloherPseudoMersenneDoubleBoundary.lean; referenceMultiplierFirstDoubleFailureProbability_lt_two_pow_neg_40; referenceMultiplierFirstDoubleFailureProbability_sum_lt_two_pow_neg_31 VQ/Experiments/SchrottenloherPseudoMersenneFailure.lean</code>	Success-domain doubling action and per-index/402-index-sum bounds for the combined event	Circuit action: odd modulus, strict modulus fit, reduced initialized operand, zero workspace, low-width bound, and pseudo-Mersenne-double success. Probability: secp256k1 primality, terminal trace, first reconstruction failure at the selected index in the missed-reduction or discarded-carry class, and the uniform reference law.	Tests/SchrottenloherPseudoMersenneDouble.lean; Tests/SchrottenloherPseudoMersenneSecp256k1.lean
Alg. 8	<code>paperLocalGates_controlled_addsMod VQ/Experiments/SchrottenloherAlgorithm8Semantics.lean; canonicalCircuit_refines_algorithm8; canonicalCircuit_act; secp256k1_program_width; secp256k1_program_toffoliCount VQ/Experiments/SchrottenloherAlgorithm8Resources.lean</code>	Five-stage circuit and canonical resource circuit produce the exact controlled modular sum and clean work. The isolated secp256k1 program has width 773 and 10,767 Toffoli gates.	Positive fitting modulus; reduced initialized source and target; initialized control; clear work field; exact-adder interface	Tests/SchrottenloherAlgorithm8.lean
Alg. 9	<code>circuit_act_of_success VQ/Experiments/SchrottenloherEfficientControlledAddSemantics.lean; referenceMultiplierFirstEfficientAdditionFailureProbability_lt_two_pow_neg_39 VQ/Experiments/SchrottenloherGenericPrimeAlgorithm9Secp256k1Bridge.lean</code>	Success-domain efficient controlled-add action and a per-index bound for the fixed efficient-model arithmetic/cleanup failure event	Circuit action: modulus fit; reduced initialized source and target; initialized control; zero reduction flag and work field; exact-adder interface; kept-width bound; controlled-add success. Probability event: secp256k1 primality, terminal trace, successful efficient reconstruction prefix and doubling, controlled-add failure, and the uniform reference law.	Tests/SchrottenloherEfficientControlledAdd.lean; Tests/SchrottenloherGenericPrimeAlgorithm9Secp256k1Bridge.lean

Item	Exact declaration and source	Proposition checked	Selected premises	Test module
Alg. 10	circuit_act_of_success VQ/Experiments/SchrottenloherPseudoMersenneControlledAddSemantics.lean; referenceMultiplierFirstAdditionArithmeticAfterDoubleProbability_lt_two_pow_neg_40 VQ/Experiments/SchrottenloherPseudoMersenneFailureProbabilityRational.lean; referenceMultiplierFirstAlgorithm10CleanupAfterPriorSuccessProbability_lt_two_pow_neg_40_via_genericPrime VQ/Experiments/SchrottenloherGenericPrimeAlgorithm10Secp256k1Bridge.lean	Success-domain pseudo-Mersenne controlled addition and two conditional per-index bounds	Circuit action: strict modulus fit; reduced initialized source and target; initialized control; zero work field; exact-adder interface; low-width bound; $0 < kept \leq n$; pseudo-Mersenne controlled-add success. Probability events: terminal trace and first reconstruction failure at the selected index; the arithmetic class excludes double failure, and the cleanup class excludes double and arithmetic failure; secp256k1 primality and the uniform reference law.	Tests/SchrottenloherPseudoMersenneControlledAdd.lean; Tests/SchrottenloherGenericPrimeAlgorithm10Secp256k1Bridge.lean
Alg. 11	circuit_act_of_success VQ/Experiments/SchrottenloherPseudoMersenneControlledAddEqualitySemantics.lean; algorithm11Secp256k1ClosedControlSplitFailureProbability_lt_two_pow_neg_38; algorithm11Secp256k1AllIndicesFailureProbability_lt_two_pow_neg_29 VQ/Experiments/SchrottenloherGenericPrimeAlgorithm11Secp256k1EventBridge.lean	Equality-aware success-domain action; prioritized suffix-after-prior-success per-index and 402-index-union bounds	Circuit action: strict modulus fit; reduced initialized source and target; initialized control; zero work, correction, and equality fields; exact-adder interface; low-width bound; $0 < kept \leq n$; equality-aware circuit success. Probability event: eligible terminal input, first reconstruction failure at the index, successful doubling and equality-aware arithmetic, enabled equality policy, one of the three prioritized suffix failures, secp256k1 primality, and the uniform reference law.	Tests/SchrottenloherPseudoMersenneControlledAddEquality.lean; Tests/SchrottenloherGenericPrimeAlgorithm11Secp256k1EventBridge.lean
Alg. 11 audit interface	no_disabledEqualityPrefixFiberData VQ/Experiments/SchrottenloherGenericPrimeAlgorithm11CoefficientFiberCounterexample.lean	At $p = 13$, the proposed control-agnostic disabled-equality prefix-fiber data do not exist	The attempted audit interface's fixed small-prime parameters	Tests/SchrottenloherGenericPrimeAlgorithm11CoefficientFiberCounterexample.lean
Schrottenloher's Table 1 baseline	sourcePointCosts VQ/Experiments/SchrottenloherTable1StaticSourceEvidence.lean; capturedGroupedTotals_matchStaticSource VQ/Experiments/SchrottenloherTable1CapturedEvidence.lean; baselineCertificates_applyRecurrence; baselineCertificates_matchCapturedRows VQ/Experiments/SchrottenloherTable1PeakWidthEvidence.lean	Static point-cost equations equal captured grouped totals. Recorded baseline root events satisfy the Qarton recurrence and captured rows.	Transcribed pinned-source constants, captured integers, and recorded events	Tests/SchrottenloherTable1StaticSourceEvidence.lean; Tests/SchrottenloherTable1CapturedEvidence.lean; Tests/SchrottenloherTable1PeakWidthEvidence.lean
Schrottenloher's Table 1 full window	sourceFullWindowPointCosts; sourceWindowGroupedIncrement_eq; sourceWindowIncrement_exceedsPaperBy VQ/Experiments/SchrottenloherTable1StaticSourceEvidence.lean; capturedExecutionRecords_groupedTotals_matchStaticFullWindowCosts; capturedExecutionRecords_fullWindowWidths_eq_baselinePlusFifteen; capturedExecutionRecords_rootPeakRecurrence_withSeventeenOutsideQubits VQ/Experiments/SchrottenloherTable1FullWindowCapturedEvidence.lean	Static formulas give the four full-window totals and increment 201,410, exceeding the paper surcharge by 4,802. Widths are baseline plus 15. Root events satisfy the recurrence with 17 qubits outside the child.	Captured schema-v2 full-window records and static pinned-source arithmetic	Tests/SchrottenloherTable1StaticSourceEvidence.lean; Tests/SchrottenloherTable1FullWindowCapturedEvidence.lean
Schrottenloher's Table 2 source	windowSchedule_partition VQ/Experiments/SchrottenloherTable2ScheduleEvidence.lean; sourceCounts_eq; sourceSpecialSpace_roundsTo2611; sourceSpecialGate_roundsTo2579; sourceGenericSpace_roundsTo2667; sourceGenericGate_roundsTo2633; sourceSpecialGate_doesNotRoundTo2578; sourceGenericSpace_doesNotRoundTo2666 VQ/Experiments/SchrottenloherTable2StaticSourceEvidence.lean	The schedule leaves 28 additions. Exact full-source totals round to 26.11, 25.79, 26.67, and 26.33. The disputed exact values are 25.79 and 26.67. The paper prints 25.78 and 26.66.	$n = 256$, $w = 16$, and exact full-window source counts	Tests/SchrottenloherTable2ScheduleEvidence.lean; Tests/SchrottenloherTable2StaticSourceEvidence.lean
Schrottenloher's Table 2 Babbush	appendixACoreGateBounds_eq; appendixAQubitBounds_eq; lowQubitPointGateBound_roundsTo2136; lowGatePointGateBound_roundsTo2100; lowQubitAppendixACoreGateBound_roundsTo2627; lowGateAppendixACoreGateBound_roundsTo2594 VQ/Experiments/SchrottenloherTable2BabbushEvidence.lean	Babbush v2 point-addition bounds and Schrottenloher's core formulas equal the stated gate and qubit integers. The gate values round to 21.36, 21.00, 26.27, and 25.94.	The premises are the transcribed v2 bounds and formulas. The audit did not inspect circuits or the Zenodo deposit.	Tests/SchrottenloherTable2BabbushEvidence.lean
Schrottenloher's Table 3	specialReconstruction_matchesPaper; genericReconstruction_matchesPaperExceptModularSquare; genericReconstruction_modularSquare_eq VQ/Experiments/SchrottenloherTable3CapturedEvidence.lean	Audit-inferred map matches all special cells and eight generic cells. The generic square reconstructs as 11.	Captured denominators/numerators and audit-attributed class/rounding map	Tests/SchrottenloherTable3CapturedEvidence.lean
Statistics: binomial definitions	binomialPointMass_eq_measure; binomialLowerTail_eq_sum_measure; binomialLowerTail_eq_measure; oneSidedPValue_eq_measure VQ/Experiments/SchrottenloherBinomialAudit.lean	The audit's finite sums agree with Mathlib's binomial measure and one-sided lower-tail p-value.	Natural trial and failure counts, with probability in $[0, 1]$ where required	Tests/SchrottenloherBinomialAudit.lean
Statistics: zero failures	companionZeroFailureWeight_eq; companionZeroFailureWeight_gt_one_third; companionZeroFailureWeight_lt_half Tests/SchrottenloherSampling.lean	At failure rate $1/10000$, the 10,000-trial zero-failure weight equals $(9999/10000)^{10000}$ and lies strictly between one third and one half.	IID Bernoulli model expressed by the imported repetition measure	Same file
Statistics: checkpoints	hasExactLength; #7 guard checks Tests/SchrottenloherBinomialAuditCheckpoints.lean	All 19 recorded failures and all 18 manifest checkpoints have the transcribed exact Euclidean length.	Fixed secp256k1 trace and transcribed values	Same file

The $p = 13$ row corrects the audit's attempted control-agnostic coefficient-fiber lemma for Algorithm 11. The counterexample refutes that lemma under its fixed small-prime parameters. The final bound uses the control-split interface indexed above.

B Counterexamples and deterministic alternative

B.1 Trace longer than 402 rounds

The checked nonzero secp256k1 residue

$$x = 52c7d5143f4c806315b9f9f28fbd6c071873e6c89587b06f9e9f8ee1d7e0cad2 \quad (2)$$

leaves the full-width Euclidean state $(1, 7)$ after 402 transitions. The state is nonterminal at that point and reaches $(1, 0)$ at transition 405. The checked declarations are `longTraceInput_after_402`, `longTraceInput_fails_402`, and `longTraceInput_succeeds_405` in `Tests/SchrottenloherPseudoMersenneSecp256k1.lean`.

This witness disproves universal termination within 402 rounds. Under the uniform-nonzero law, the displayed input contributes probability $1/(p-1)$. Schrottenloher’s Table 1 aggregate numerical bound still requires a denominator count or another valid distributional argument together with the other row-level events and their dependence structure.

B.2 Tight bound and 512-round layout

The declarations `euclidStep_product_double_le` and `euclidTrace_product_bound` prove a product-shrink invariant. The sharp 511-round result uses `euclidStep_bitLengthSum_lt` and `euclidTrace_steps_add_final_bitLengthSum_le`: every live step strictly reduces $\text{bitLength}(u) + \text{bitLength}(v)$, and the terminal pair retains bit-length sum one. The coprime and prime corollaries `traceSucceeds_two_mul_bits_sub_one_of_coprime` and `traceSucceeds_two_mul_bits_sub_one_of_prime` specialize this descent to the $2n - 1 = 511$ bound. The input $x = 2^{255}$ is at $(1, 1)$ after 510 rounds and first reaches the terminal pair at round 511, as checked by `powTwo255_after_510`, `powTwo255_fails_510`, and `powTwo255_succeeds_511`. Thus 511 is the tight universal bound for this full-width transition.

The corrected reversible circuit executes 512 decisions so that the terminal transition and fixed padding have a uniform schedule. Its exact-adder theorem `correctedCircuit_act` covers every nonzero reduced input and reduced target with the work and record registers initially clean. The theorem proves the modular product and endpoint cleanup under those premises.

The corrected schedule requires a new resource calculation. Its compressor needs

$$\left\lceil \frac{512}{3} \right\rceil = 171 \quad \text{blocks and} \quad 171 \cdot 5 = 855 \quad \text{record bits.}$$

The two reported 374-bit shared fields provide $2 \cdot 374 = 748$ positions, so $855 > 748$. The declarations `deterministicRecord_exceeds_paperFields` and `no_paperFieldWidth_scheduleLegal` check the storage incompatibility. The corrected layout’s whole point-addition cost remains open.

B.3 Round-369 shrinking-field witness

The publication shrinking schedule has the checked input

$$x = \text{ac2ad9731956a0318967f615ff0085ca6c38d615de6ffcf3e390f25d79000854}. \quad (3)$$

After 369 rounds its full state is $(6033964351, 44029)$. The effective comparison, whose width is the smaller of the active field width and 77, agrees with the full comparison and selects the same ordered odd branch. The declarations `domainFailureState_after_369`, `domainFailureState_padding_fails_369`, and `domainFailureState_comparison_agrees_369` in `Tests/SchrottenloherArtifactTrace.lean` check the full state, padding failure, and comparison agreement.

The agreeing branch produces different next successors in the two transition models:

$$\text{full} = (44029, 3016960161), \quad \text{clipped} = (44029, 869476513).$$

The declarations `domainFailureState_algorithm2Step_369`, `domainFailureState_artifactStep_369`, and `domainFailureState_step_mismatch_369` check those values and their inequality. The next update loses high bits from a live operand and violates `ArtifactTraceDomain`. The preceding comparison remains correct.

Under the uniform law on $\{1, \dots, p-1\}$, one failing input gives a probability contribution exactly $1/(p-1)$. The theorem `one_div_p_sub_one_le_valueFailureProbability` in `VQ/Experiments/SchrottenloherArtifactTraceFailure.lean` applies that one-point argument to the shrinking trace model. The number of additional shrinking failures and the aggregate with termination, comparison, arithmetic, and cleanup remain unresolved.

C Resource arithmetic and class map

C.1 Captured reporter output

Table 10 transcribes the eight canonical schema-v2 result JSON files in paper row order. Width is the captured `Circuit.nbr_qubits()` output. Bits is the captured `Circuit.nbr_bits()` output. Raw CCZ is zero in every row and is omitted from the table. Adding raw CCX and forward AND gives the displayed grouped total. Measured uncompute is the count of zero-weight measured-AND uncomputations under the grouped convention. Basic sum adds every entry returned by `basic_gate_counts()`.

Table 10: Raw baseline and full-window captured reporter values.

Mode	Width	Bits	Raw CCX	Fwd. AND	Meas. uncompute	Grouped	Basic sum
Baseline special space	1,192	1,541	1,740,787	644,730	526,144	2,385,517	19,929,384
Baseline special gate	1,445	1,541	502,635	1,362,886	1,360,076	1,865,521	31,170,396
Baseline generic space	1,191	1,701	2,430,771	1,177,272	698,494	3,608,043	35,852,486
Baseline generic gate	1,446	1,541	1,325,731	1,477,788	1,474,978	2,803,519	37,862,666
Full special space	1,207	1,556	1,742,547	844,380	725,794	2,586,927	66,002,343
Full special gate	1,460	1,556	504,395	1,562,536	1,559,726	2,066,931	77,243,355
Full generic space	1,206	1,716	2,432,531	1,376,922	898,144	3,809,453	81,925,445
Full generic gate	1,461	1,556	1,327,491	1,677,438	1,674,628	3,004,929	83,935,625

The exact baseline grouped exponents, rounded to the nearest hundredth in the paper’s order, are 21.19, 20.83, 21.78, and 21.42. These values reproduce the four displayed baseline non-Clifford entries. The reproduction is a static arithmetic and captured-output result under the pinned cost convention. The total-gate and failure-probability columns remain unresolved.

C.2 Static schedule-source arithmetic

The static source evidence transcribes the publication schedule as 402 rounds with padding 37, compressed-record width 670, coordinate width 374, and remaining padding 78. It also checks adder budget 183, active-width sum 60,124, and retained-comparison-width sum 27,775. Pinned schedule-source formulas and exact Lean arithmetic give these quantities.

The same source evidence derives `ToBitVector` grouped costs 219,735 in space mode and 212,569 in gate mode. Together with the other transcribed component formulas, those values produce all four baseline and full-window grouped totals. The baseline totals are checked by `SchrottenloherTable1StaticSourceEvidence.sourcePointCosts`. The full-window totals and surcharge are checked by `sourceFullWindowPointCosts`, `sourceWindowGroupedIncrement_eq`, and `sourceWindowIncrement_exceedsPaperBy` in the same module. These declarations establish the grouped-count arithmetic. Python liveness and Schrottenloher’s Table 1 total-gate aggregation require separate derivations.

C.3 Root events and SWAP observation

A root-event tuple orders its coordinates as $(oldPeak, newCurrent, childPeak, oldCurrent, previousInside)$. Applying Equation (1) produces the final column. The child peaks are unchanged between baseline and full-window runs. The root’s live-current increment changes the result.

Table 11: Captured root-event tuples and recurrence outputs.

Mode	Event tuple	Peak
Baseline special space	(1031, 514, 1190, 514, 512)	1,192
Baseline special gate	(1031, 514, 1443, 514, 512)	1,445
Baseline generic space	(1031, 514, 1189, 514, 512)	1,191
Baseline generic gate	(1031, 514, 1444, 514, 512)	1,446
Full special space	(1057, 529, 1190, 529, 512)	1,207
Full special gate	(1057, 529, 1443, 529, 512)	1,460
Full generic space	(1057, 529, 1189, 529, 512)	1,206
Full generic gate	(1057, 529, 1444, 529, 512)	1,461

Qarton’s asymmetric quantum/classical SWAP update is in `qarton/circuit/memory_manager.py`, lines 529–542, at revision 7506faba2147 [6]. In the source labels, `qc` means that operand zero is quantum and operand one is classical. The affected branch removes operand zero from the quantum set and adds operand zero again, leaving operand one absent from the transfer. Instrumentation recorded zero `qc` cases, zero affected-branch cases, and zero incorrect transitions in all eight canonical results. The recorded width discrepancies arise elsewhere.

C.4 Surcharge and Schrottenloher’s Table 2 calculations

The full-window grouped increment expands directly from the pinned formulas:

$$\begin{aligned}
3(2^{16} - 2) &= 196,602 && \text{forward lookup,} \\
3(1528) &= 4,584 && \text{charged gates in measured unlookup,} \\
4(56) &= 224 && \text{nonzero tests,} \\
196,602 + 4,584 + 224 &= 201,410 && \text{total.}
\end{aligned}$$

The paper’s $3 \cdot 2^{16} = 196,608$ differs by 4,802. The address grows from one bit to 16 bits, which agrees with the captured width increment $16 - 1 = 15$.

Table 12: Exact core arithmetic for Schrottenloher’s Table 2.

Mode	Paper rounded $\log_2$	28(baseline + 196,608)	Its $\log_2$	Full total	Full $\log_2$
Special space	26.11	72,299,500	26.11	72,433,956	26.11
Special gate	25.78	57,739,612	25.78	57,874,068	25.79
Generic space	26.66	106,530,228	26.67	106,664,684	26.67
Generic gate	26.33	84,003,556	26.32	84,138,012	26.33

The second numeric column equals 28 times the exact baseline grouped count plus the paper’s surcharge inside the parentheses. The full total is 28 times the exact full-window grouped count. Both formulas omit the initial direct lookup, QFT resources, and measurement resources.

C.5 Babbush substitutions

The two checked comparison substitutions are

$$\begin{aligned}
28(2,700,000 + 3 \cdot 65,536) &= 81,105,024, & 1,175 + 16 &= 1,191, \\
28(2,100,000 + 3 \cdot 65,536) &= 64,305,024, & 1,425 + 16 &= 1,441.
\end{aligned}$$

Rounding base-two logarithms gives point-addition entries 21.36 and 21.00 and core entries 26.27 and 25.94. These calculations transcribe published Babbush v2 bounds and Schrottenloher’s composition. Babbush et al. withheld the circuits. The inspected evidence excludes the public Zenodo computation.

C.6 Complete inferred map for Schrottenloher’s Table 3

The special and generic denominators are 2,385,517 and 3,608,043. Each cell in Table 13 lists selected class numerators followed by the integer percentage obtained after Qarton’s two-decimal per-class rounding and summation. Names ending in `_inv` are separate captured recursive classes.

Table 13: Audit-inferred recursive-class map for Schrottenloher’s Table 3.

Component	Special-prime selection: numerators $\rightarrow$ result	Generic-prime selection: numerators $\rightarrow$ result
Modular square	<code>ControlledSpecialPrimeModularSquareAdd_inv</code> : 207,964 $\rightarrow$ 9	<code>ControlledEfficientModularSquareAdd_inv</code> : 380,314 $\rightarrow$ 11 (paper 10)
In-place multiplier and inverse	<code>IPModMul</code> , <code>IPModMul_inv</code> : 1,080,986 each $\rightarrow$ 45 + 45 = 90	Same names: 1,606,074 each $\rightarrow$ 45 + 45 = 90
Bézout reconstruction	<code>ApplyBitVector</code> , <code>ApplyBitVector_inv</code> : 641,516 each $\rightarrow$ 27 + 27 = 54	Same names: 1,166,604 each $\rightarrow$ 32 + 32 = 64
GCD construction	<code>ToBitVector</code> , <code>ToBitVector_inv</code> : 439,470 each $\rightarrow$ 18 + 18 = 36	Same names: 439,470 each $\rightarrow$ 12 + 12 = 24
Controlled swap	<code>ControlledSwap</code> : 446,320 $\rightarrow$ 19	Same name: 446,320 $\rightarrow$ 12
Hybrid adder	<code>ControlledHybridAdder</code> , <code>ControlledHybridAdder_inv</code> : 254,828 each $\rightarrow$ 11 + 11 = 22	Same names: 254,828 each $\rightarrow$ 7 + 7 = 14
Controlled modular adder	<code>SpecialPrimeControlledModularAdder</code> : 375,584; inverse 541,216 $\rightarrow$ 16 + 23 = 39	<code>EfficientControlledModularAdder</code> : 699,078; inverse 922,310 $\rightarrow$ 19 + 26 = 45
Modular double	<code>SpecialPrimeModularDouble</code> , inverse: 107,340 each $\rightarrow$ 4 + 4 = 8	<code>EfficientModularDouble</code> , inverse: 434,859 each $\rightarrow$ 12 + 12 = 24
Gidney constant adders	<code>GidneyConstantAdder</code> : 3,835; <code>ControlledGidneyConstantAdder</code> : 350,304 $\rightarrow$ 0 + 15 = 15	Same names: 3,835; 1,237,920 $\rightarrow$ 0 + 34 = 34

The generic modular-square ratio is $380314/3608043 = 10.540728\dots$ percent. The exact boundary calculation $200(380314) - 21(3608043) = 293897 > 0$ places it above 10.5 percent, so the inferred rule yields 11. All other inferred cells equal the paper’s values, giving 17 agreements among 18 cells.

`SpecialPrimeHandlePControlledModularAdder` and its inverse each have numerator 68,550 and rounded contribution 3 percent. Adding both changes the total for the controlled modular adder from 39 to 45. The 45-percent

selection and the printed 39-percent selection are both compatible with the retained captures because the authors’ class map is absent.

D Statistical and execution provenance

D.1 Statistical inference and run identity

The statistical execution used manifest repository_revision a72e2e28aa5da256e8b0015288e6cdce5aeb6d0, which differs from the formal-evidence revision 00ce72179927. Its manifest records CPython 3.13.5 on aarch64 Linux 6.12.57+deb13 with glibc 2.41, sampling through `os.getrandom` without flags from 19:35:07 to 19:35:26 UTC on 18 August 2026. It stopped after 1,000,000 accepted candidates, rejected zero, recorded 19 failures, maximum length 415, 361,842,246 total Euclidean steps, and a 32,000,000-byte raw transcript.

For the composite null $q \geq 1/10100$ under the IID-uniform accepted-residue model, the one-sided exact-binomial p-value is enclosed by

$$\leq P \leq \frac{8.3798656826905476271985335220787562621312046249779483847264991343122804735246240 \times 10^{-23}}{8.3798656826905476271985335220787562621312046249779483847264991343122804736932623 \times 10^{-23}}.$$

The one-sided 95-percent Clopper–Pearson endpoint lies between

$$\frac{35340977961419490592736597}{1267650600228229401496703205376} \quad \text{and} \quad \frac{17670488980709745296368299}{633825300114114700748351602688},$$

whose decimal enclosures are

$$0.0000278791158660412077778946104658, \\ 0.0000278791158660412077778946112548.$$

The inference JSON records the displayed p-value and Clopper–Pearson interval endpoints. Section 5.3 reports the upper endpoint as 2.79×10^{-5} . The declarations `binomialPointMass_eq_measure`, `binomialLowerTail_eq_sum_measure`, `binomialLowerTail_eq_measure`, and `oneSidedPValue_eq_measure` in `VQ/Experiments/SchrottenloherBinomialAudit.lean` identify the finite sums with Mathlib’s binomial measure. The test `Tests/SchrottenloherBinomialAudit.lean` checks the exact 5/16 lower tail for one failure in four fair trials and the integer threshold $2^{133} < 10100^{10}$. The file `Tests/SchrottenloherBinomialAuditCheckpoints.lean` defines `hasExactLength` and checks all 19 failures and all 18 manifest checkpoints against their exact Euclidean lengths. For the paper’s 10,000-test observation, `companionZeroFailureWeight_eq`, `companionZeroFailureWeight_gt_one_third`, and `companionZeroFailureWeight_lt_half` in `Tests/SchrottenloherSampling.lean` check the exact zero-failure weight and its strict one-third and one-half bounds at failure rate 1/10000.

Table 14: Tracked statistical files and identities.

Record	SHA-256 or identity
Protocol specification file	277dfa9340593a7ae6520c1e356c4d898d24e70fff39425d5915566004af3651
Result document	893d7530f489c61f316295988b971deccb6c16720025996834dbe3d1112b0123
Manifest file	f51f7ead862ced632be24553ae304863e5cb4063010e9fc53ba9202001bb86e6
Inference file	9cc187d00b448d9a1ad3731c6cbb0b5f7c99f9769901549003b29c0780c19662
Failures file	20bdf793f7ae9b9c0820b7a08d09816da8c198ba3d0c07defb4991d524c9fd07
Manifest-recorded protocol hash: protocol bytes and path absent	18fc4f3654092393391caac5677ef446275fcbe723ad6347a357c662ce74786c
Audit tool	cf7dccc351195a6f0f2b7a2827c77ccea5092bceaadf5e838d547c337ee0f4b9a
Transcript checker	08d123afbc6c07de80fbb3dcee6e1ef7ae3feb649fd9a84820c9ca8b28568b7
Raw transcript	fcdbf7beb91a20289b2debb50479fd9bc0a446db231d9099737b08d7e95ecfe

The five tracked statistical files reside under `paper/`. The raw path was `tmp/schrottenloher-tail-iid-20260818/candidates.bin`. The raw transcript has no durable URI. The recorded SHA-256 digest identifies the raw bytes used during checking. The entropy source and IID premise depend on the recorded execution. The retained protocol and manifest have no independent preregistration timestamp.

D.2 Failures and fixed checkpoints

Table 15 is the complete tracked failure record. Accepted index gives the ordinal among accepted residues. Each hex value has a Lean-checked trace length, and the transcript checker matched its index and value to the raw bytes available during the audit.

Table 15: All accepted residues with trace length above 402.

Index	Input hexadecimal	Steps
1,603	f330b51486064c75c9dfbb33799267f2e1d04c75a9084dd6e018ffa7155fa6bc	403
72,384	ee97610141334d630dbaf33719415f0b2ff0a01ea5947b10457aa5690cab3a	409
112,203	ca514b4002c0281524f4f88efb7b49f9c9d7071f995eb2e21a15f4c5f274c39	405
119,539	fad382c49e2472f0b694d87c07489b40a7df44b147d1629ab707419ba5e0670a	406
184,962	e9db47ba7c4973054370c787977a4ee72f6f12c63c331ca882d4391d35f3a97	408
316,336	d1ee090ef28dac9cd60153577259bbfb1c04d15e12feed9ce0b0b904fcec638	403
445,667	7b8f194b0c4f9db150547d2f080aa47402034ecbdec5530b853dd700807ed68	405
534,197	17c113033b4c2d13a6432c37beb305cb078ca9ecdad64499c8c250c7e477a3364	409
539,833	7a2ad96bf835d793a1b13e042ca3f42dc32f99473d4d2a9bebe3dc2175a737c	415
540,966	718a1a9ec060dd270ae3bbcc302df44e3030ad456c6176fb0cbda55846a59b10	404
548,934	a898e0782e13a26a0eda6aeae8dbf807f51aaf4d6d806c644101881da3cd49fc46	403
575,265	e8b785e63fa70280211f0aa9934bb38af2f777b2d0eale75b2565adfc2dc8f2a	406
610,405	916f733cb95dedf1ff57ea987a86d344b5e7088f6ce9a3a78256564369ecb49e	407
687,025	c8f6140b82d6e27ac8725a8bf9499b01a06f6c4749744688564464a4479878b	406
784,238	4b8f31880c9ae48156767d20d40866c46e4d46ab65056f8f3cef3552d8f2c29e	403
791,939	3a58052878f7b29003177a2182b1349f30843a699f9ca471bd297a5f469c9e	405
828,678	1c730e2ad3924563c779af3a00d81f8ae28e0090314bce405d8a3327f1f52459	406
888,266	f913cdeb0499d79ebe28fcc7e18a1da83de95542776cf738944b3a4891bia20	407
973,930	818b1770d542c5f1fabcc64f003966e417e829bffe3be3dd3d9aba39838f4	404

The manifest embeds the 18 checkpoints in Table 16. The first eight check the start of the accepted sequence, and the remaining ten check each 100,000th accepted position. The transcript checker used these checkpoints to test fixed accepted-sequence positions while the raw bytes were available.

Table 16: Fixed checkpoints embedded in the statistical manifest.

Index	Input hexadecimal	Steps
1	b4edc433e7dd99f3bdf3797d236c44d254b4c477ab94ae605ea94f6442dd2882	355
2	bd757c0f2b6b831773759b8dd8e69e48c23872c2d9aa54f7bad5da82a109c505fc	371
3	10c048a90966a2eda14aa0cb8e628bed5b9cf03936057d63b56ef5ce49673cef	366
4	6d8a7a785c47770d63dc38c1494e177089f3f81753d557aab1856732c064d9d	369
5	e943ad80247b598d102593a20cf129c1cd63aebb4959cc0d61d5f825da341067	358
6	df88cc60335bd431c4508271a595a5ed04b204d6298f70b837e2c1dcfa9a273c	356
7	41ec8176ca882cc04208544f71d8b3c0fba0269e0ada338f6c7f4082a79c1c54	374
8	8890da9ae56c277f8b4bc0c1ed84f9aceb700eb2bb0ba812684956b95a90ded2	361
100,000	4f75b606170514bcae4f372cb574e318f8eaa081dfa02bd045684d9ea9c3a14	377
200,000	48eba7f08c111fa69a6a29b7bc458150191b33710a7dbf6716538152c26b4ef	365
300,000	4ae8426fb73aa5b964f0e1431ed82f2aa64feca36782887eb99c017d6683f6	364
400,000	897bec5d22c69f1f7b47c45df57baea5fe116c0ec290a151db0ccde5e9794bb	370
500,000	61faf31eb7e2e18345242c06c222ed5fab29e990ff83ab56c136b74dd85b2830	366
600,000	cd9a865712d36b5079627d1ce9a83f0a2050b184de0afdf76fb4dce455859497	364
700,000	330cbbe983275aee6d8016ac0950800443c9b46a5b37cfa56558eb4fa764c44	354
800,000	d164ec3e740188066442839822f4b709d293240d8f739a1ac4b7ad4aa1704f7	361
900,000	9fe5328da3f2bd1d734055f3442d0b682b2c1f12513c820bc36a498eb16da3	354
1,000,000	0d3d6d38da8672336377e868ad325fd226e847d68d4b89cec5bf791e2e3688f9	365

D.3 Resource capture identities

The four resource suites ran on 21 August 2026 UTC, corresponding to the evening of 20 August in the audit’s local time. They used uv 0.10.2, CPython 3.12.12 invoked with -I -B, Qarton 0.2.1, companion revision 3b60d5050b18, and Qarton revision 7506faba2147. All mode processes returned status zero with empty standard error, and the two results for each record class were byte-identical.

Table 17: Resource manifest, tool, and source identities.

Object	SHA-256
Baseline manifest 1	3a383fe577ffa7eb3f292438a14c453364d5bb651531db3ca1901feba9e8c198
Baseline manifest 2	04c358b7d21fe8c77d99047ffc50015f3050552828c41416c27689368734c02f
Full-window manifest 1	6dae9923110b6ec084c7bc4c62337f240a8bf724cfb35128748f9d994c28e2c
Full-window manifest 2	0b6186d69d60f93e9ede5fcd063d21e7d0ea1af9fc77f98d36a11dba25c55cd
Capture program	03eab7cfce28bd3af68f060b3e4d0f35a65725a87f33a0d6ac5cf821ead5a3aa
Capture test	820dd423b0406d60a5ed6cca417ce36e7e1df9f0212a333157acd4f5770f6594
Four-mode runner	bed00916e1c18d3e8480587ecce6ab3685fd35525207e6a2d50d5ae30c53c981
Runner test	67aa46d984621eadb49194a13dab5531ce8d23b18490e1d8ad81466fcc30a8d3
Full table digest	85d3a20ffa586ffcc144d125f0a662246ccba82afec84a9905e3ec25ddde380
Companion source archive	74fb7bb4c691b49317227c9960393c39c3dfa7365345e98bf24813a8bd6b40ca
Qarton source archive	708e0ad1b0bd1f47019fc4abee78f82ad0e8622a6730dd0cd7cac8934bf892bf
Bundled publication PDF	18f80b0757839aa925f791e3bfe37462b29685fffd8e355d347e5176e4ec9b1d
Companion 15-file Python tree	d303ee4ec339d77c019ae3e454ffbf6f408294d15ad61422ed05673561d863a7d
Qarton 297-file Python tree	a48164ad760c5744b9d0b22c26d96d586806bcde6f794b12f6cb4588c4323b82
Companion build_circuit.py	a12d4f57df9dea777f1ae718bea48aa63ae91685742f108df2c64c1b3bcf14f3
Installer	feef1a6b1e055dff5e52d23441c6a50e231ee941acac23943aa8690c214f615
Qarton pyproject.toml	ec10aad30fd62b8b98878b0128692a4a8bf44babfc6c52796516a8dda399e3b

The bundled publication PDF’s byte identity with the arXiv PDF remains unchecked. The retained manifests record hashes of archives, source trees, and a PDF inspected during capture. The evidence repository contains those manifests, result JSON files, and the capture README. The hashed archives, source trees, and PDF are absent. The revision labels state the intended Git provenance. The evidence record contains no authenticated link from the inspected bytes to a remote repository at the named revision.

Table 18: Canonical result-file and payload hashes.

Mode	Baseline file / payload	Full-window file / payload
Special space	b14baf18b8a4bd50d00ab92c07b0ee8b5404c394802b9d9d65a28ece7163d5 / bc71e7d696559f0e97926ffbaa2539032efffa74585ccc7868f6a53f35e8dc3	4881266462b656e8b7655168543c1505f9521fcea964a380f6af2f9d22cd55be / 0885a810a190a4f5660229e50ad7b82893b224fcc950ababdf6c74422cb45c50
Special gate	6e734152bc395a47df8228e9e546f2af7ec409e14b383b2e290e1602ab53e08 / 27084e7bf0f01a23766a61caf0e82a8545db41b0e6d94e503b9115a8792af984	c921a301c83ae09c9f8a2a4553f4d3d5d34f1e3f348bc673e5e450588ddb7c1 / 5431accbaf69dc9b67f2e76272641995f4f5f0a117c879adcc8aec26c28ae170
Generic space	69adac20271d99af5cd075cb4b9f559ee33c8f6c03d30bfdec45ba2608251b4d / ea599e3204152377ce75381e0eb6c903f6f5aebc43ff8c5b724f5fec145c15cd	f160baca524ceab6684e2391ade356910e11e7e15de104046642c9abc7c0b661 / 371a56232beab9292456c650596ba89ea7ac5bd7fb39c18d2c67b2e3383d19e8
Generic gate	bf1db744ec4f0851ca4b34068c2e8bccc9fbd1e25e89a473781bc3b6293fdd82b / 0ecfa4119f37b5095fc298633ca4449038938c238c0b0a55d017847fb7948789	brd1ade981b8bcf36143ca43691c2daaab8221733a457c47eebf062f0e3dbe4 / b549e7a99964c6ff48b75482ea8ba4cb222b189f36a541e7b5c0be4d90cd5a08

The SHA-256 values record digests for the retained files and the recorded directory walk. The repeated outputs establish byte identity across two executions. Python semantics, circuit cleanup, physical qubit requirements, and Git-host provenance require separate evidence.

D.4 Resource-capture software environment

The schema-v2 execution environment used the distributions and versions in Table 19. Qarton came from the pinned archive in the virtual environment. The isolated commands placed separate clean Qarton and companion source roots first on `sys.path`.

Table 19: Installed resource-capture distributions and versions.

Package	Version	Package	Version	Package	Version
antlr4-python3-runtime	4.13.2	numpy	2.5.2	rich	15.0.0
contourpy	1.3.3	openqasm3	1.0.1	scipy	1.18.0
cycler	0.12.1	packaging	26.3	setuptools	84.0.0
fonttools	4.63.0	pillow	12.3.0	six	1.17.0
frozendict	2.4.7	pip	26.2.1	sympy	1.14.0
iniconfig	2.3.0	pluggy	1.6.0	tabulate	0.10.0
kiwisolver	1.5.0	Pygments	2.21.0	tqdm	4.70.0
markdown-it-py	4.2.0	pyparsing	3.3.2	types-tabulate	0.10.0.20260508
matplotlib	3.11.1	pytest	9.1.1	types-tqdm	4.70.0.20260805
mdurl	0.1.2	python-dateutil	2.9.0.post0	typing_extensions	4.16.0
mpmath	1.3.0	qarton	0.2.1	wheel	0.48.0

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