

A Formal, Statistical, and Source Audit of Schrottenloher’s Optimized Point-Addition Circuits for Elliptic-Curve Discrete Logarithms

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Abstract

Schrottenloher proposes quantum point-addition circuits whose resource savings depend on a compressed binary-Euclidean trace, reversed Bézout reconstruction, and approximate modular arithmetic. We audited arXiv:2606.02235v1 through Section 2, Algorithms 1–11, Figure 1, and Tables 1–3 using Lean 4 proofs, exact arithmetic, statistical records, and executions of pinned publication source. The formal results establish the compressed trace and compressor circuit, conditional reconstruction and in-place multiplication, exact modular doubling and controlled addition, and success-domain semantics for the approximate operations. The audit finds a secp256k1 residue whose trace exceeds the 402-round schedule and terminates at round 405, proves the tight 511-round universal bound for the full-width trace, verifies a deterministic 512-round multiplier schedule, and gives a round-369 witness against the shrinking-field value model. The arithmetic-event bounds proved under a uniform law on nonzero Euclidean inputs and independent reduced coefficients remain conditional because no theorem transfers that law to the paper’s point-addition call states or combines every failure class. The grouped counts derived from the pinned source reproduce all four displayed baseline non-Clifford exponents in Schrottenloher’s Table 1. Captured peak widths disagree with two printed widths, while static arithmetic over the pinned full-window source gives a surcharge of 201,410 units rather than the stated 196,608 units. Exact source arithmetic changes two displayed base-two exponents in Schrottenloher’s Table 2 by 0.01, while the audit-inferred map for Schrottenloher’s Table 3 assigns 11 percent, rather than 10 percent, to generic modular squaring. These results verify the named component statements and document the discrepancies and conditional claims, but do not establish the paper-wide failure probability or whole elliptic-curve discrete-logarithm resources.

1 Scope, models, and evidence

1.1 Audited construction and dependency chain

The audited object is Schrottenloher’s first arXiv version, arXiv:2606.02235v1 [10]. The elliptic-curve discrete logarithm problem (ECDLP) asks for the scalar d satisfying $Q = [d]P$ in a cyclic elliptic-curve group. The paper’s sixteen claim groups are Section 2, Algorithms 1–11, Figure 1, and Tables 1–3, with the following dependency order:

ECDLP recovery $\longrightarrow$ windowed scalar multiplication $\longrightarrow$ Algorithm 1 point addition
 $\longrightarrow$ Algorithm 4 multiplication $\longrightarrow$ Algorithms 2–3 and Figure 1
 $\longrightarrow$ Algorithms 5–11.

The paper’s quantum reduction evaluates two scalar multiplications in superposition and then applies quantum Fourier transforms (QFTs), which are unitary discrete Fourier transforms on quantum registers. Any missing premise in the arithmetic oracle limits downstream recovery and composite-resource claims that use that oracle.

Arithmetic value, reversible-circuit action, work- and record-register cleanup, failure probability under a named law, and support for a publication resource count are distinct propositions. A value identity does not prove that ancillas return to zero, and a successful source execution does not prove a law over all circuit inputs. Each disposition below names the proposition that its evidence supports.

The evidence dispositions used below have fixed meanings. A *formal result* is a proposition checked by Lean at the cited revision, while a *conditional result* retains explicit domain, success, or probability-law premises that the paper execution has not discharged. A *checked counterexample* is a Lean-checked witness against either a paper

claim or an attempted audit lemma, with the target identified. A *source reconstruction* is exact arithmetic over quantities or formulas transcribed from pinned publication source, and a *source execution* is recorded output from that source under a named environment. A *statistical result* depends on its stated sampling model and provenance, while *unresolved* means that the paper, cited sources, retained code, and audit results do not contain the evidence needed for the claim.

1.2 Parameters and circuit terms

The fixed field modulus is the secp256k1 prime

$$p = 2^{256} - 4294968273,$$

with field width $n = 256$ and scalar-window width $w = 16$. Schrottenloher writes q for this modulus. This report uses p to agree with the formal records. The windowed construction treats two 256-bit scalars, giving 32 window ordinals before schedule adjustments.

A CNOT gate flips one target bit when one control bit is set. CCX denotes the two-control NOT, or Toffoli, and CCZ denotes the two-control phase-flip gate. The source reporter also counts a temporary logical AND: its forward computation receives the charged non-Clifford cost, while a measurement-based uncomputation is recorded separately at zero cost under the reporter’s grouped convention. The grouped non-Clifford count is therefore raw CCX plus raw CCZ plus charged forward temporary AND. All retained resource captures have raw CCZ count zero, so their grouped count equals raw CCX plus forward AND.

IID means independent and identically distributed. Independence is not assumed unless a result names it, since different calls inside one reversible computation share state. An IID premise occurs in the external million-residue audit, not in the formal proof that one Euclidean transition or one modular operation has the stated action.

For this report, VQ means the Lean evidence repository at revision 00ce72179927 [13]. Its relevant statements interpret reversible gates on basis-state bit registers and prove exact register action under their displayed premises. Separate statements address cleanup, finite probability, and resource arithmetic, so the aggregate build cannot add an unstated connection among those layers.

1.3 Sources, records, and probability law

The publication evidence consists of the paper, companion revision 3b60d5050b18 [8], schedule-source revision d8d2473cb43c [9], and Qarton revision 7506faba2147 [7]. Formal statements, theorem tests, resource arithmetic, counterexamples, and statistical record checks come from VQ revision 00ce72179927. The retained source and statistical records are listed below [12, 11]. All paths are relative to that revision.

Captured resource directories:

`paper/schrottenloher-resource-capture/schema-v2-baseline`

`paper/schrottenloher-resource-capture/schema-v2-full-window`

Source-analysis reports:

`paper/schrottenloher-402-source-audit.md`

`paper/schrottenloher-resource-audit.md`

Statistical files:

`paper/schrottenloher-tail-statistical-audit.md`

`paper/schrottenloher-tail-statistical-audit-result.md`

`paper/schrottenloher-tail-statistical-audit-manifest.json`

`paper/schrottenloher-tail-statistical-audit-inference.json`

`paper/schrottenloher-tail-statistical-audit-failures.json`

The 18 checkpoints are entries in the manifest, not a separate record. The external comparison uses Babbush et al. v2 [2], its public Zenodo deposit with program, outputs, verification keys, and Groth16 proofs [1], and the original Clopper–Pearson interval construction [4]. Appendix D records the statistical hashes and execution boundary.

The *reference law* draws the Euclidean input uniformly from $\{1, \dots, p - 1\}$ and, independently, draws the reconstruction coefficient uniformly modulo p . Earlier formal records call this same distribution the *reference-product law*. The two names are identical here, and the shorter name is used below. A probability theorem under

this law says nothing about the distribution induced by the paper’s basis-label registers until a bridge or event-specific comparison proves the transfer.

An *accepted nonzero residue* is a 32-byte big-endian candidate interpreted as an integer x with $1 \leq x < p$. A *valid-block readout* reads a valid raw six-bit triple after forward compression as its compressed value below 2^5 , with the sixth wire clear. The reverse action decodes that value. A *captured root event* is the five-tuple recorded when the child class `IPModMul_inv` is appended to the root `WindowAffAddSpaceOpt`: old peak, new current, child peak, old current, and the number of previous qubits inside the mapped child. Qarton’s peak-width recurrence at revision 7506faba2147 is

$$\max(\text{oldPeak}, \text{newCurrent}, \text{childPeak} + \text{oldCurrent} - \text{previousInside}). \quad (1)$$

The retained resource records apply this recurrence to a captured event. They do not derive the event trace from Python syntax.

1.4 Audited models

The full-width mathematical trace supplies the transition and termination reasoning without field clipping. The publication-source shrinking schedule uses 402 rounds, padding 37, comparison-truncation parameter 40, and field width 374. Its checked comparison width is $\min(\text{currentWidth}, 77)$. The checked 402-round pseudo-Mersenne multiplier instead uses padding 40, retains 40 comparison bits, and uses equality-aware addition in the first 50 forward rounds. The deterministic corrected multiplier fixes 512 rounds and enlarges the record layout.

Table 1: Models and checked evidence.

Model	Value and circuit semantics	Cleanup	Termination or probability	Resource evidence
Full-width mathematical trace	Parameterized step, trace, compression, and reconstruction semantics	Trace reversal and reconstruction on stated domain	Universal 511-round bound. Tight witness.	Compression-block count only
Publication shrinking schedule	Conditional schedule-circuit action and every compressed record block under trace-domain, padding, odd-left, and clean-register premises	Conditional raw/work cleanup under the same premises. No total multiplier theorem.	Round-369 domain/value witness and one-witness uniform-nonzero lower bound	Pinned schedule-source formulas
Checked 402 pseudo-Mersenne multiplier	Exact basis-state multiplier action on trace, padding, and approximate-arithmetic success premises	Checked on that success domain	Checked 405-round counterexample to universal termination. Reference-law component bounds.	Isolated formal counts, not a composition for Schrottenloher’s Table 1
Deterministic 512 multiplier	Corrected fixed-schedule multiplication action with the exact adder	Checked for every nonzero input and reduced target under clean-work and register premises	Universal termination from the tight 511-round theorem	Record layout checked. Whole point-addition cost not derived.

The models share the full-width binary-Euclidean transition when all retained values fit and the selected approximate operations satisfy their success predicates. Their schedules, padding, active widths, and some arithmetic stages differ. Results below name the applicable model, and no result is transferred between rows of Table 1 without a checked bridge.

1.5 Disposition of the sixteen claim groups

Table 2: Disposition of the sixteen paper claim groups.

Item	Paper-facing statement	Disposition	Corrected statement or unresolved premise	Sec.
§2	One observation recovers the ECDLP with probability greater than one half.	Formal result and unresolved	The ideal Fourier-state reference theorem is checked. A generated oracle, its induced state, and the paper-execution probability theorem are absent.	2.1
Alg. 1	Windowed affine point addition supplies the oracle arithmetic.	Conditional result	The register path and affine-result formula are checked. Separate enabled and disabled value lemmas retain group-validity and noncollision premises or zero-selected-point and representability premises. Exceptional paths, concrete tables, persistent selection state, a measured-unlookup realization, composite resources, and the call-state law are absent.	2.3
Alg. 2	A fixed 402-round binary-Euclidean schedule records the choices and is used with the asserted high-probability bound.	Formal result, checked counterexample, and unresolved	Parameterized trace semantics are checked, but one residue disproves total 402-round termination. The claimed tail remains unproved. Every nonzero input terminates by 511 rounds, and that bound is tight.	3.1, 3.5–3.6, 5.2–5.3
Fig. 1	Three Euclidean choices compress reversibly from six bits to five retained bits.	Formal result	Forward and inverse actions, valid-block readout, and the 13-gate cost are checked. Reconstruction’s decoder handles a partial final block.	3.2–3.3
Alg. 3	Reversing the choices reconstructs the Bézout coefficient.	Formal result and conditional result	Pure compressed reconstruction is checked from a terminal trace. The generated approximate reverse schedule separately requires successful selected coefficient steps.	3.3
Alg. 4	Trace and reconstruction form an in-place modular multiplier.	Conditional result, formal result, and checked counterexamples	The checked 402-round pseudo-Mersenne multiplier is conditional on its success domain. A fixed 512-round version gives deterministic termination with a larger record. The separate publication shrinking schedule has a round-369 value failure.	3.4–3.7
Alg. 5	Five stages perform exact modular doubling.	Formal result	Stage refinement, exact value action, and work-register cleanup are checked for odd modulus and stated width premises.	4.1
Alg. 6	Truncated generic-prime doubling has a small failure event.	Conditional result	Success-domain circuit action and a per-index reference-law bound are checked.	4.3, 5.4
Alg. 7	Pseudo-Mersenne doubling has a small failure event.	Conditional result	Success-domain action and reference-law bounds for the combined missed-reduction/discarded-carry event are checked.	4.3, 5.4
Alg. 8	Five stages perform exact controlled modular addition.	Formal result	Stage refinement, exact controlled value action, and cleanup are checked under the theorem’s input conditions.	4.1
Alg. 9	Truncated generic-prime controlled addition has a small failure event.	Conditional result	Success-domain action and a reference-law result for the combined arithmetic/cleanup event are checked.	4.4, 5.4
Alg. 10	Pseudo-Mersenne controlled addition has small arithmetic and cleanup events.	Conditional result	Separate reference-law results retain their named prior-success conditions. The audit does not assume independence.	4.4, 5.4
Alg. 11	Equality-aware controlled addition handles $x + y = p$.	Conditional result	Circuit action, three prioritized failure classes, and per-index and 402-index reference-law results are checked. The fixed checked multiplier sets <code>equalityIterations=50</code> . Grouped baseline counts reproduce. Two widths and the full-window surcharge disagree. Total-gate aggregation and the paper-wide failure probability remain unsupported.	4.5, 5.4
Paper T1	Four point-addition rows have the printed widths, grouped costs, total-gate values, and failure bound.	Source reconstruction, source execution, and unresolved	The factor 28 and core arithmetic reproduce, but exact full-window-source arithmetic changes two displayed exponents by 0.01. Babbush rows are checked arithmetic over published bounds, not whole-circuit results.	5, 6.2–6.5
Paper T2	The point-addition rows compose to the displayed ECDLP exponents, including two Babbush comparisons.	Source reconstruction	The factor 28 and core arithmetic reproduce, but exact full-window-source arithmetic changes two displayed exponents by 0.01. Babbush rows are checked arithmetic over published bounds, not whole-circuit results.	6.6–6.7
Paper T3	The listed classes account for the printed percentage breakdown.	Source reconstruction and unresolved	The audit-inferred map matches 17 of 18 cells. Generic modular squaring is about 10.54 percent and rounds to 11. The authors’ class map and rounding order are absent.	6.8

2 ECDLP reduction and point addition

2.1 Section 2 reference result

The checked Section 2 endpoint is a reference theorem for secp256k1 ECDLP recovery from an ideal circuit state. For a valid encoded point $Q = [d]P$ and sufficient semantic level for the exact positive QFTs, the theorem identifies the emitted terminal state, proves normalization and the scalar-pair marginal, and proves recovery of d on a selected observation set. If r is the secp256k1 subgroup order, the selected mass is at least

$$\frac{r-1}{r} \frac{2401}{14641},$$

which is approximately 0.164 and does not imply a probability above one half. The Lean theorem names this internal order constant q . The report uses r to avoid conflict with the paper’s field-modulus notation. Its recovery

procedure decodes the two Fourier outcomes, uses a modular inverse when the required coefficient is nonzero, and validates the recovered public point. The proposition is `VQ.Tests.ECDLPFourierRecovery.secp256k1_reference_verification`. Its source and assumptions appear in Appendix A.

That reference result does not identify Schrottenloher’s generated point-addition oracle with the ideal arithmetic oracle. Such an identification requires a complete generated oracle, a circuit theorem for its induced state before the QFTs, and a probability theorem relating the paper’s measurement to recovery. The audited sources contain no composition that supplies those three components, so the paper’s statement that one observation recovers the logarithm with probability greater than one half is unresolved at the implementation level.

Standard ECDLP reduction mathematics cannot fill this circuit gap. The mathematical reduction assumes the group action used to form the periodic or coset state, while Algorithm 1 is the proposed implementation of that action. A theorem about the ideal state proves what the Fourier measurement would do after correct oracle preparation. It does not prove that the proposed arithmetic circuit prepares that state.

2.2 Input-law obstruction

The paper starts from two uniform 256-bit basis-label registers, a finite source with $2^{256}2^{256}$ equally weighted labels. The reference law has $(p - 1)p$ equally weighted pairs, because its nonzero Euclidean input has $p - 1$ choices and its independent coefficient has p choices. If a deterministic function pushed the uniform basis-label law exactly to the uniform reference law, every target point would have a fiber of the same integer size. Hence $(p - 1)p$ would divide 2^{512} .

The checked cardinality theorem rules out that divisibility. The odd prime p divides $(p - 1)p$ but is coprime to every power of two, so it cannot divide 2^{512} . The formal statement `algorithm0BasisLabel_not_pushesForward_reference` therefore proves that no deterministic map from the two uniform 256-bit labels has the exact reference-law pushforward.

The obstruction rules out one transfer. Reference-law bounds for the multiplier’s internal pairs cannot transfer to the paper’s basis-label law by an exact pushforward identity. An event-specific domination inequality, a coupling with controlled error, or a direct analysis of Algorithm 1 call states could still establish useful bounds, but no such argument occurs in the retained evidence. Section 5 therefore keeps every reference-law result conditional.

2.3 Algorithm 1

The published generic affine path of Algorithm 1 has checked whole-register action and an affine-result formula under explicit premises. The `selector16_point_addition_act` theorem covers the generated sequence of lookups, modular subtraction, in-place division, controlled squaring, modular addition, negation, and final unlookup on its register layout. The companion `selector16_point_addition_affine` theorem rewrites that register result as `affineResult` with control `decide(address ≠ 0)` when the selected shift equals $3 \text{ selected}X \bmod p$. These statements prove the complete written-register result for the path, including the zero values required in its work regions, but do not identify that result with a group operation.

The action and affine theorems require zero point, lookup, subtraction, and division work regions; reduced selected and target coordinates; a reduced selected shift with the required table value; unequal selected and target x coordinates; and a positive adjusted division input. Separate lemmas cover the two control values. The theorem `selectedPoint_algorithm1_true` identifies the enabled affine result with `Curve.addPoint` under `GroupValid`, unequal- x , and recovery-noncollision premises. The theorem `selectedPoint_algorithm1_false` proves preservation only when the selected coordinates are $(0, 0)$, the target is representable, and its x coordinate is nonzero. The theorem `selectedPoint_groupAdd` gives a separate group-valued identity for `groupAddValue` under `GroupValid`. No checked theorem composes these value lemmas with the generated circuit into a total group action.

Several obligations needed by the paper’s whole oracle lie outside those theorems. The audited result does not cover equal- x exceptional cases, point at infinity, or a complete circuit-to-group branch dispatcher. It does not instantiate and validate the publication’s concrete point and shift tables, prove cleanup of persistent branch-selection state, attribute measured-unlookup circuitry that the paper does not specify, derive the composite point-addition resource row, or determine the law of arithmetic arguments over the superposed scalar-multiplication run. The downstream audit may therefore use the whole-register affine-result formula and the separate value lemmas only under their displayed premises.

3 Euclidean trace, compressed record, and multiplier

3.1 Algorithm 2: transition and trace circuit

Algorithm 2 applies a binary-Euclidean transition to a pair of nonnegative remainders beginning at (p, x) . Each stored choice has an **odd** bit for the current right remainder and a **swap** bit for the ordered odd case. Validity requires that a set swap bit accompany a set odd bit. The transition halves an even right remainder or uses the ordered odd case to replace and halve the relevant difference. The terminal pair for a nonzero residue modulo the prime is $(1, 0)$, after which a fixed schedule can retain terminal padding choices.

The parameterized compressed-trace circuit has separate checked statements for its three observable parts. Starting from bounded input registers and clean carry, sign, comparison, and scratch wires, its input-pair theorem equals the mathematical pair after the requested number of transitions. Its record theorem identifies every five-bit compressed block with the mathematical choice record, while its cleanup theorem returns the four temporary decision wires to zero. These results hold for a parameter **rounds**. They do not prove that the paper’s specialization **rounds = 402** reaches $(1, 0)$ for every nonzero secp256k1 input.

The publication shrinking schedule also has a conditional generated-circuit theorem. In `VQ/Experiments/SchrottenloherArtifactSchedule.lean`, `paperScheduleGates_action` proves the artifact-trace active pair, raw/work cleanup, and every compressed record block under `ArtifactTraceDomain`, odd initial left operand, final `PaddingSafeAt`, and clean raw, work, and record premises. `paperScheduleCircuit_wellFormed` checks circuit well-formedness. This is a schedule-circuit action theorem rather than a total multiplier theorem.

The trace and record claims also have different failure modes. A too-short schedule may record 402 correct transitions and still stop before the terminal pair. A shrinking register may select the correct comparison and then alter the next value by clipping it. Sections 3.5 and 3.7 give checked witnesses for these two distinct failures.

3.2 Figure 1: three-choice compression

One Euclidean choice is a pair of Boolean values subject to **swap** implying **odd**, leaving three valid choices. A block is an ordered triple of choices encoded on six wires, two wires per choice. There are $3^3 = 27$ valid six-bit blocks, which fit in five bits. Valid-block readout means that, for any such block, the circuit’s six-bit output equals the compressed code and is below 2^5 , so its sixth wire is clear and its retained five bits determine the original triple.

The Figure 1 gate list has checked forward and inverse action on all 27 valid inputs. Copies placed on consecutive blocks have checked family readout and reverse to the original family input. Separately, reconstruction’s `decodeCompressedRecord` handles a final partial block containing one or two valid choices rather than assuming that every schedule length is divisible by three. Each full block uses exactly 13 gates: 5 CCX gates, 6 CNOT gates, and 2 single-bit NOT gates.

The paper’s 402 decisions occupy $402/3 = 134$ blocks and retain $134 \cdot 5 = 670$ bits. A deterministic 512-decision schedule occupies $\lceil 512/3 \rceil = 171$ blocks, including one partial block, and retains $171 \cdot 5 = 855$ bits. The two layouts therefore use the same checked compressor but have different record requirements.

3.3 Algorithm 3: reversed Bézout reconstruction

The pure reconstruction theorem follows an invariant between the current coefficient pair and the exact Euclidean remainder pair. Each reverse choice updates the coefficients by the algebraic inverse corresponding to its forward transition. Reading the compressed record backward reproduces the uncompressed choice sequence, including a final block containing one or two valid choices. If the recorded trace from (p, x) is terminal and $y < p$, the checked pure result maps the coefficient pair to $(0, xy \bmod p)$ for odd p .

This pure result has a terminal-trace premise and no approximate-arithmetic success premise. Its theorem `reconstructCompressed_compressedTraceCircuit_of_terminal` connects the record emitted by the compressor circuit to the mathematical reconstruction. The fixed $2n$ corollary derives terminality from primality, a positive nonzero residue, and the full-width $2n$ schedule.

The generated reverse schedule is a separate circuit result. Its coefficient registers have finite padding, and its selected modular operations may use truncated or pseudo-Mersenne approximations. The generated schedule maps $(y, 0)$ to $(0, xy \bmod p)$ and cleans its temporary registers only when every named coefficient-step success predicate holds for the recorded trace. Those premises belong to the circuit and composite multiplier, not to the pure compressed reconstruction theorem.

3.4 Algorithm 4: conditional multiplication

The in-place multiplier composes three reversible stages. The forward trace generates the remainder pair and compressed choices, the reverse reconstruction consumes those choices while transforming the multiplier target, and the inverse trace erases the Euclidean record and restores the input register. On a terminal trace with adequate padding and successful selected approximate operations, the checked 402-round pseudo-Mersenne circuit implements

$$(x, y) \mapsto (x, xy \bmod p)$$

and returns its record and work registers to their initial clean values.

The domain premises describe the theorem’s exact limit. It accepts any $x < 2^{256}$ and $y < p$ for which the 402-step trace is terminal and the pseudo-Mersenne reconstruction success predicate holds. The convenience corollary packages those conditions as a reference-multiplier success predicate, but does not prove that every input satisfies it. Neither the multiplier theorem nor its local resource bounds establish Algorithm 1’s complete affine path, its input law, or a point-addition total in Schrottenloher’s Table 1.

3.5 Origin of the 402-round schedule

The paper models the binary-Euclidean iteration count with empirical mean $1.413n$, observed standard deviation about $0.6\sqrt{n}$, and schedule parameter $c_{\text{iter}} = 2.4$. The schedule source at revision `d8d2473cb43c` evaluates

$$3 \left\lceil \frac{1.413n + 2.4\sqrt{n}}{3} \right\rceil = 402$$

for $n = 256$, rounding to a multiple of three for the compressor. The cited binary-Euclid analysis labels the product-shrink and normal-tail steps used for this choice as heuristics [3]. Neither the paper nor that source supplies a fixed-denominator theorem for the secp256k1 residues whose traces exceed 402 rounds.

An empirical schedule supports the claimed probability only if its tail is bounded under the law induced by the full algorithm. Here the termination law is also unresolved: the paper does not derive the distribution of multiplier inputs inside Algorithm 1, and Section 2 rules out one proposed exact pushforward to the reference law. The statistical audit in Section 5.3 addresses uniform accepted nonzero residues under an IID premise. It does not bound the tail under Algorithm 1’s induced call law.

3.6 Checked counterexample and deterministic correction

The 402-round specialization is not universally terminating in the checked pseudo-Mersenne trace model. One residue leaves the pair $(1, 7)$ after round 402 and reaches $(1, 0)$ at round 405. Appendix B gives the complete residue and theorem locations, so this correction does not depend on a statistical search record.

For every live full-width transition, Lean separately proves the product inequality $2u'v' \leq uv$. The sharp termination proof instead uses strict descent of $\text{bitLength}(u) + \text{bitLength}(v)$ at every live step. Its trace inequality, with coprimality of (p, x) and the terminal bit-length sum, gives the universal $2n - 1 = 511$ round bound for nonzero secp256k1 residues. The input 2^{255} first terminates at round 511, showing that the 511-round upper bound is attained. A fixed 512-round reversible schedule with the exact adder therefore has a checked multiplication and cleanup theorem for every nonzero input and reduced target under clean-work and register premises. It has no approximate-arithmetic success premise.

The deterministic correction changes the layout. Its 171 compression blocks retain 855 record bits, while the paper’s two shared 374-bit Euclidean fields provide 748 positions. The inequality $855 > 748$ is a checked incompatibility with that reported shared-field allocation. The corrected multiplier terminates deterministically. It does not preserve or repair the paper’s resource rows without a new layout and cost derivation.

3.7 Shrinking-field failure

The publication shrinking schedule has a value failure independent of the 402-round tail. For one checked residue, round 369 begins from $(6033964351, 44029)$. The full comparison and the comparison on the retained bits choose the same branch, yet the full successor is $(44029, 3016960161)$ and the clipped successor is $(44029, 869476513)$. A correct branch decision therefore does not prevent a live operand from losing high bits before the next arithmetic update.

This witness concerns the publication-source shrinking model with 374-bit field allocation, 37 padding bits, comparison-truncation parameter 40, and effective checked comparison width $\min(\text{currentWidth}, 77)$. The witness

violates the **ArtifactTraceDomain** premise of the conditional schedule theorem. It is not the 405-round witness and does not rely on a failed comparison. Under the uniform law on nonzero residues, the existence of the one displayed input gives the shrinking trace model’s value-failure probability the checked lower bound $1/(p - 1)$. Appendix B records the complete input and divergent state data.

4 Exact and approximate modular operations

4.1 Algorithms 5 and 8: exact operations

Algorithm 5’s five stages implement exact modular doubling and clean their work registers for an odd modulus. The stages shift the widened target, load and subtract the modulus, conditionally add it back and unload the modulus, invert the subtraction flag, and cancel that flag with the low result bit. The stage lemmas refine the pseudocode values into one register theorem, including the equivalence between the paper’s cleanup order and the order in the selected reversible realization. Under $x < p$, $p \leq 2^n$, clean source and flag registers, and odd p , the final target is $2x \bmod p$ and all non-target registers return to their input values.

Algorithm 8’s five stages implement exact controlled modular addition and clean their work registers under its stated input conditions. Its refinement first conditionally adds the source to the target, subtracts p in a widened register, conditionally restores p , recomputes the comparison information needed to clear the borrow, and inverts the final ancilla. For reduced $x, y < p$, a clean workspace, and a Boolean control, the checked result is y when the control is false and $(x + y) \bmod p$ when it is true. The theorem also proves that the widened high bit and arithmetic work region are clear at the endpoint.

These results connect the pseudocode stage order to chosen reversible circuits, their register placement, and their compiled primitive gates. The associated isolated counts are facts about those implementations. They do not derive the publication’s composite point-addition rows, which also depend on lookup, inversion, squaring, branch handling, and the source reporter’s cost convention.

4.2 Success domains and failure classifications

A success-domain circuit theorem proves the intended register action and cleanup when named arithmetic predicates hold. It does not assign a specified wrong output to inputs outside the domain. Separate value-level classification lemmas locate particular predicate failures in intervals or arithmetic cases. Their conclusions apply only to the failure class and model named in each lemma.

Table 3 records the checked semantic scope before any numerical probability statement. Each probability result uses the reference law and appears in Section 5.4. No row supplies the distribution of Algorithm 1 call states.

Table 3: Circuit semantics and named events for approximate operations.

Alg.	Intended operation	Checked circuit result	Named failure classes	Probability result
6	Generic-prime modular doubling	Writes $2x \bmod p$ and cleans the selected workspace when the efficient-double success predicate holds.	Selected high-part-gap comparison event.	Per-index reference-law theorem. See Sec. 5.4.
7	Pseudo-Mersenne modular doubling	Writes $2x \bmod p$ and cleans the low-width workspace when the pseudo-Mersenne success predicate holds.	Missed reduction and discarded carry.	Per-index and 402-index-sum reference-law theorems. See Sec. 5.4.
9	Generic-prime controlled modular addition	Writes the controlled sum and cleans its workspace on the efficient-add success domain.	Combined arithmetic and cleanup event for the generic construction.	Per-index reference-law theorem. See Sec. 5.4.
10	Pseudo-Mersenne controlled modular addition	Writes the controlled sum and cleans its workspace on the pseudo-Mersenne success domain.	Addition arithmetic after successful doubling. Cleanup after prior success.	Two conditional per-index reference-law theorems. See Sec. 5.4.

Alg.	Intended operation	Checked circuit result	Named failure classes	Probability result
11	Equality-aware pseudo-Mersenne addition	Implements the equality-aware controlled sum and cleans its workspace when the equality-aware success predicate holds.	Detection, correction cleanup, and equality cleanup, in priority order.	Per-index control-split and 402-index-union theorems. See Sec. 5.4.

4.3 Algorithms 6 and 7: approximate doubling

Algorithm 6 uses only a retained high part to decide whether doubling crosses the modulus. The efficient-double circuit theorem states exact modular doubling and cleanup when the high-part comparison agrees with the reduction needed by the full value. A generic-prime probability module classifies a selected disagreement as a high-part-gap event, and the secp256k1 bridge identifies that generic event with the event used by the fixed circuit model. This bridge connects two event definitions. It does not connect the reference law to the point-addition oracle.

Algorithm 7 exploits the pseudo-Mersenne relation between p and 2^{256} . Its low-width computation can fail by missing a required modular reduction or by discarding a carry that affects the retained result. The circuit action is checked whenever neither event occurs, and separate classification lemmas describe the value-level failure intervals. The law and numerical bounds for their combined event are stated in Section 5.4.

The two doubling algorithms have different event definitions even when their intended value is the same. Algorithm 6’s high-part gap belongs to the generic comparison model, whereas Algorithm 7’s event uses the pseudo-Mersenne correction and carry structure. A bound for either event cannot be assigned to an Algorithm 1 division or addition call without a theorem matching both the call state and the event.

4.4 Algorithms 9 and 10: approximate controlled addition

Algorithm 9 has checked success-domain action for generic-prime controlled addition. Its selected failure event combines an arithmetic discrepancy with the cleanup condition needed to restore the work register. The coefficient-fiber argument bounds that combined event under the reference law, but the event is neither Algorithm 6’s high-part gap nor the pseudo-Mersenne event used by Algorithm 10. The numerical result remains in Section 5.4 to keep circuit action and law-dependent inference separate.

Algorithm 10 has checked success-domain action for pseudo-Mersenne controlled addition. Its evidence separates the arithmetic event for an addition performed after a successful doubling from the cleanup event after all named prior operations succeed. The two bounds retain different conditioning statements, and the audit does not assert that the events are independent. Their use in a composite probability argument would require the same conditional order as the circuit schedule.

The control wire is part of each operation’s semantics. When the control is false, the target remains unchanged while the work registers still have to clean. When it is true, the circuit must produce the reduced sum. Treating the two control fibers as one unqualified coefficient fiber can lose the premise needed for cleanup, a problem that becomes explicit in Algorithm 11.

4.5 Algorithm 11: equality-aware addition

Algorithm 11 adds equality handling to the pseudo-Mersenne controlled operation used in the first 50 forward rounds of the checked multiplier. Its circuit theorem implements the intended equality-aware sum and cleanup on the named success domain. The failure classification orders three events: detection fails to recognize the exact equality condition, correction cleanup leaves information after the equality correction, or final equality cleanup leaves a low-block residue. Priority prevents one input from being charged under a later class after an earlier class has already occurred.

The final probability proof splits on the control. The disabled branch and enabled branch have different coefficient-fiber structures, so each receives the event definition supported by its algebraic premises before the results are recombined. A checked $p = 13$ counterexample showed that the audit’s first control-agnostic fiber interface could not exist. It refutes that attempted audit lemma, not a statement in Schrottenloher’s paper. Appendix A identifies the counterexample theorem and the exact interface it targets.

5 Failure-probability analysis

5.1 Events and probability laws

The probability attached to Schrottenloher’s Table 1 combines at least five event families: failure to terminate within 402 rounds, invalidity introduced by shrinking live fields, a wrong branch from truncated comparison, approximate modular arithmetic failure, and incomplete cleanup. The 405-round witness concerns the first family, while the round-369 witness concerns the second despite a correct comparison. The Algorithm 6, 7, 9, 10, and 11 theorems concern specific arithmetic or cleanup events. No checked statement identifies their union with the paper’s complete circuit-failure event.

All arithmetic bounds below use the reference law defined in Section 1. The paper begins instead with the two-register basis-label law, and Algorithm 1 produces a sequence of dependent arithmetic calls inside scalar multiplication. The audit has no theorem deriving the reference law for those calls and no event-specific domination theorem that bounds every named failure under their induced law. It therefore makes no independence assumption among calls or among conditional events.

A paper-wide probability result would need one probability space for the complete computation. Within that space, each circuit failure class would need a measurable event, and conditional bounds would have to follow the execution order in which their premises become available. Summing local reference-law values without such a bridge would change both the law and the event being bounded.

5.2 Zero failures in 10,000 tests

The paper reports 10,000 random tests with no observed failures and prints $2^{-13.3}$ in Schrottenloher’s Table 1 caption. The schedule source at revision `d8d2473cb43c` contains the simulation procedure and uses deterministic worker seeds, but the publication bundle does not archive the sampled inputs or its output transcript. The binomial reading below is conditional on IID Bernoulli trials. The retained source does not establish that premise. The reported zero count is not an exhaustive circuit test.

The later statistical protocol selects $1/10100$ as a rational null boundary. Lean checks $1/10100 < 2^{-13.3}$ through the integer inequality $2^{133} < 10100^{10}$. The comparison below therefore tests a rational rate that is already stricter than the captioned decimal exponent.

Suppose IID trials had true failure probability $1/10000$. The probability of seeing zero failures would be

$$\left(\frac{9999}{10000}\right)^{10000},$$

which the audit bounds between one third and one half. The rate $1/10000$ exceeds both $1/10100$ and $2^{-13.3}$, yet the reported zero count occurs with probability in that interval.

The one-sided 95-percent Clopper–Pearson upper endpoint after zero failures in 10,000 trials is

$$1 - 0.05^{1/10000} \approx 3.00 \times 10^{-4}$$

under the binomial model [4]. This endpoint exceeds both $1/10100$ and $2^{-13.3}$. The observed test result therefore does not establish the captioned upper bound at that confidence level, and it supplies no universal theorem about the generated circuit.

5.3 Million-residue termination audit

The external termination audit samples 32-byte big-endian candidates and accepts a candidate exactly when its integer value x satisfies $1 \leq x < p$. Under the stated premise, accepted nonzero residues are IID and uniform on $\{1, \dots, p-1\}$. Its failure classifier is $L_p(x) > 402$, where $L_p(x)$ is the full-width binary-Euclidean trace length from (p, x) .

The tracked manifest and result document record 1,000,000 accepted candidates, no rejections, 19 candidates classified as longer than 402 rounds, and a maximum length of 415. Conditional on the IID-uniform model and recorded provenance, the one-sided 95-percent Clopper–Pearson endpoint is about 2.79×10^{-5} , below $1/10100$. The observed fraction is $19/1,000,000$. The endpoint accounts for sampling uncertainty rather than treating that fraction as the true rate.

The checked and external parts of this result remain separate. Lean checks the trace lengths of the 19 listed failures and 18 checkpoints, exact binomial definitions, a rational test case, and the integer comparison placing

1/10100 below $2^{-13.3}$. A dependency-free evaluator encloses the million-trial binomial calculation, while a transcript checker compares the listed positions and values with the raw bytes. Lean does not check the IID premise, transcript provenance, composite-null monotonicity, million-trial numerical enclosure, or Clopper–Pearson bracket. The 32 MB raw transcript has no durable URI, so Appendix D reports the tracked record without claiming that the raw sample is independently retrievable.

5.4 Conditional arithmetic-event bounds

Table 4 collects the numerical results after the semantic definitions in Section 4. Every row fixes the secp256k1 modulus and uses the reference law. An index refers to the selected reverse Euclidean step in the checked 402-round model. Aggregate entries use a union or sum over those 402 indices, not an independence calculation.

Table 4: Checked arithmetic-event probability bounds under the reference law.

Alg.	Event	Range	Checked numerical result and retained condition
6	Selected generic high-part-gap event	One reverse index	At most $(2^{216} - 1)/p$. The fixed bridge equates the generic and secp256k1 circuit events.
7	Pseudo-Mersenne missed reduction or discarded carry	One reverse index and sum over 402	Below 2^{-40} per index and below 2^{-31} for the sum over 402 indices.
9	Generic controlled-add arithmetic or cleanup event	One reverse index	Below 2^{-39} for the combined event.
10	Addition arithmetic after successful doubling	One reverse index	Below 2^{-40} under the named successful-doubling condition.
10	Cleanup after prior success	One reverse index	Below 2^{-40} under the named prior-success condition.
11	Prioritized detection, correction-cleanup, or equality-cleanup event after control split	One reverse index and union over 402	Below 2^{-38} per index and below 2^{-29} for the 402-index union.

The rows have different logical forms. Algorithm 7’s aggregate is a sum of the same indexed event family, while Algorithm 11 first splits the control fibers, applies the prioritized classification, and then unions indices. Algorithm 10’s two rows are conditional events in an execution sequence. None is an unconditional failure probability for Algorithm 1 or the complete ECDLP circuit.

5.5 Schrottenloher’s Table 1 aggregate failure bound remains unproved

Four missing implications prevent a proof of Schrottenloher’s Table 1 aggregate failure probability. First, the Algorithm 1 arithmetic-call law must be derived, or each selected reference-law event must be dominated under that induced law. Second, the 402-round termination tail must be bounded under the same law, or the schedule must be replaced. Third, termination, shrinking-field, comparison, arithmetic, and cleanup failures must all be defined as events in one circuit model. Fourth, their union must be bounded in the conditional order of the computation.

The current formal and statistical results do not supply those four implications. The deterministic 512-round theorem removes the termination tail only for the larger corrected schedule, whose record does not fit the paper’s reported shared-field layout. It leaves the other event classes and Algorithm 1 input law unchanged, so it cannot be inserted into the paper’s probability or resource rows without a revised construction.

6 Resource-source audit

6.1 Source versions and execution method

The resource executions use publication companion revision 3b60d5050b18 [8]. They use Qarton revision 7506faba2147 [7]. The four schema-v2 capture suites ran in one uv-managed CPython 3.12.12 environment with Qarton 0.2.1 and the package versions listed in Appendix D. Two suites used the companion’s baseline table $(0G, G)$, and two directly instantiated the pinned point-addition class with all 65,536 rows. Every mode exited with status zero and empty standard error. Corresponding results are byte-identical across the two baseline suites and across the two full-window suites.

The four schema-v2 manifests and eight canonical result JSON files preserve source identities, package versions, commands, and output hashes for the August 2026 execution, subject to the hash and dependency limits stated

below. They cannot reconstruct the authors’ June 2026 environment because the companion gives dependency lower bounds and uses a mutable base image rather than an exact lockfile. The four manifests also record clean pre/post source states for each run.

Three evidence forms must remain separate. Static source arithmetic evaluates formulas and constants transcribed from the pinned Python source. Captured reporter output records what that Python program emitted under the retained environment. A syntax-derived theorem would prove from the program text that every execution creates the stated event trace and peak liveness. The audit has the first two forms and checked arithmetic connecting them. It has no syntax-to-event or syntax-to-liveness derivation for Python.

6.2 Schrottenloher’s Table 1 baseline counts

For these captures, the grouped metric is raw CCX plus raw CCZ plus charged forward temporary AND. Every retained result reports zero raw CCZ, so the grouped count equals raw CCX plus forward AND. Measurement-based AND uncomputation appears as a separate reporter count and receives zero grouped cost under the pinned convention. A measured-unlookup circuit still contains other charged gates, which account for its nonzero surcharge below.

Mode	Captured grouped	Captured width	Paper width
Special-prime, space	2,385,517	1,192	1,192
Special-prime, gate	1,865,521	1,445	1,446
Generic-prime, space	3,608,043	1,191	1,192
Generic-prime, gate	2,803,519	1,446	1,446

Table 5: Baseline grouped counts and peak widths in the paper’s row order.

Static source reconstruction and captured execution agree on all four grouped counts in Table 5. Two captured widths disagree with the paper: the special-prime gate mode gives 1,445 rather than 1,446, and the generic-prime space mode gives 1,191 rather than 1,192. Appendix C separates raw CCX, forward AND, measured uncompute, grouped total, and basic reporter sum for every mode.

The exact grouped counts have nearest-hundredth base-two exponents 21.19, 20.83, 21.78, and 21.42 in the paper’s row order. These values reproduce the four displayed baseline non-Clifford entries. The result concerns the pinned grouped-cost convention. It supplies neither the total-gate column nor the failure-probability column.

6.3 Peak-width evidence

The captured root events defined in Section 1 reproduce the four baseline widths through Equation (1). Each baseline event has old peak 1,031, old and new current 514, and 512 previous qubits inside the mapped multiplier. The four child peaks are 1,190, 1,443, 1,189, and 1,444 in paper row order. Thus the third recurrence term adds two live root qubits to each child peak and gives 1,192, 1,445, 1,191, and 1,446.

The full-window root events have old peak 1,057, old and new current 529, and the same 512 previous-inside qubits and child peaks. Their recurrence term adds 17 rather than two, producing 1,207, 1,460, 1,206, and 1,461. These five-tuples are retained instrumentation records at Qarton revision 7506faba2147, whose source locator for the recurrence is cited directly in [7]. Arithmetic over an event confirms the recorded width but does not prove that the Python program must emit that event.

Pinned Qarton also contains an asymmetric SWAP bookkeeping transition for the case in which a quantum/classical mapping reaches the affected branch. The schema-v2 instrumentation reports zero reached qc SWAP events, zero faulty-branch events, and zero incorrect transitions in all retained runs. The defect therefore cannot explain either baseline width disagreement in these executions [6]. Appendix C records the event tuples and the observed SWAP classifications.

6.4 Full-window construction and surcharge

The publication command builds the point-addition circuit with a two-row lookup table, so it cannot produce the 2^{16} -row window assumed in the resource discussion. The audit’s full-window runner directly invokes the pinned `WindowAffAddSpaceOpt` class with rows $([0]G, [1]G, \dots, [65535]G)$ while retaining the paper’s four choices of prime specialization and gate/space mode. The full-window records are thus pinned-source executions of the point-addition class, not literal replays of the publication command.

The source-derived surcharge has three parts. Three forward lookups cost 196,602 grouped units, charged select-swap and forward-AND gates inside three measured-unlookup circuits add 4,584, and four nonzero tests add 224. The 4,584 term does not charge the zero-weight measured-AND-uncompute events themselves. The sum is

$$196,602 + 4,584 + 224 = 201,410,$$

which exceeds the paper’s 196,608-unit surcharge by 4,802. Replacing the baseline one-bit address with a 16-bit address raises the reported peak width by 15 qubits, not 16.

Mode	Full-window grouped	Full-window width	Baseline increment
Special-prime, space	2,586,927	1,207	201,410 / 15
Special-prime, gate	2,066,931	1,460	201,410 / 15
Generic-prime, space	3,809,453	1,206	201,410 / 15
Generic-prime, gate	3,004,929	1,461	201,410 / 15

Table 6: Full-window captures and grouped-count/width increments.

All four captured grouped increments equal the source reconstruction. The measured width increments also agree with the address replacement and the captured root events. Neither agreement supplies a whole-ECDLP theorem, because the runs instantiate one point-addition class and do not generate the complete two-scalar schedule, initial lookup, Fourier transforms, or measurement procedure.

6.5 Unsupported total-gate column

The publication source does not specify which operations enter Schrottenloher’s Table 1 total-gate column. It also gives no aggregation or rounding rule from the reporter output to the printed estimates. The captured “basic sum” adds every entry returned by `basic_gate_counts()`, but no evidence identifies that sum as the paper’s rule. The schema-v2 baseline and full-window result JSON files report basic-gate dictionary sums of 19,929,384, 31,170,396, 35,852,486, and 37,862,666 for the baseline modes, and 66,002,343, 77,243,355, 81,925,445, and 83,935,625 for the full-window modes. These are reporter data, not reconstructions of the paper’s total-gate column, so that column remains unresolved.

6.6 Schrottenloher’s Table 2 schedule and arithmetic

Two 256-bit scalar registers contain $2 \cdot 256/16 = 32$ window ordinals. The schedule replaces the first point addition by a direct lookup and defers the final three additions, leaving $32 - 1 - 3 = 28$ retained point additions in the displayed core formula. That formula omits the initial direct lookup. The paper gives no numerical term that would add it to the core total.

Table 7 compares three arithmetic interpretations. The first recomputes the paper using its rounded point-addition inputs and stated 196,608 surcharge. The second uses exact baseline counts plus that paper surcharge. The third multiplies the exact full-window source counts by 28. The corrected widths come from the full-window captures, not by adding the paper’s stated 16 qubits.

Mode	Paper rounded-input $\log_2$	Exact baseline + paper surcharge $\log_2$	Exact full-source $\log_2$	Paper width	Full width
Special-prime, space	26.11	26.11	26.11	1,208	1,207
Special-prime, gate	25.78	25.78	25.79	1,462	1,460
Generic-prime, space	26.66	26.67	26.67	1,208	1,206
Generic-prime, gate	26.33	26.32	26.33	1,462	1,461

Table 7: Three calculations for Schrottenloher’s Table 2 in its row order.

The exact full-source exponents are 26.11, 25.79, 26.67, and 26.33. The second and third values correct the corresponding printed 25.78 and 26.66 by 0.01. The exact full-source totals are 72,433,956, 57,874,068, 106,664,684, and 84,138,012. Appendix C gives the parallel baseline-plus-paper-surcharge totals and every substitution. This is a formula reconstruction over one point-addition cost, not a syntax-derived resource theorem for the complete ECDLP circuit.

6.7 Babbush comparison rows

Babbush et al. report average executed CCX-plus-CCZ point-addition bounds of 2.7 million in the low-qubit construction and 2.1 million in the low-gate construction, with respective point-addition qubit bounds 1,175 and 1,425 [2]. Their reported average point-addition bounds derive from 9,024 hash-selected test cases. Schrottenloher’s Table 1 displays those bounds after rounding as point-addition exponents 21.36 and 21.00. Equations (A1)–(A3) in Babbush et al. separately compose the published point-addition quantities into their core estimates.

Substitution into Schrottenloher’s 28-addition comparison gives

$$\begin{aligned} 28(2,700,000 + 3 \cdot 2^{16}) &= 81,105,024, & 1175 + 16 &= 1191, \\ 28(2,100,000 + 3 \cdot 2^{16}) &= 64,305,024, & 1425 + 16 &= 1441. \end{aligned}$$

Their base-two gate exponents are 26.27 and 25.94, reproducing Schrottenloher’s displayed comparison rows. The formal evidence checks these integer substitutions and the associated rounding.

The circuits used for the Babbush et al. estimates are withheld. Their public Zenodo v2 deposit has DOI 10.5281/zenodo.19597130. It contains the program, public outputs, verification keys, and Groth16 proofs [1]. This audit did not verify that bundle. The published equations give no numerical upper bound for the initial direct lookup, lookup uncomputation, or recycled-QFT work. The reconstructed values are therefore checked core arithmetic over published point-addition bounds, not exact totals for a released whole circuit.

6.8 Schrottenloher’s Table 3 class reconstruction

Qarton’s recursive class reporter records costs at both parent and child classes and uses separate names for forward and inverse classes. The paper does not publish the class-selection map, overlap policy, or rounding order used for Schrottenloher’s Table 3. The audit infers a rule that selects named captured classes, applies Qarton’s `round(numerator/denominator, 2)` to each selected recursive class, and then sums the corresponding integer percentages. This rule is an audit reconstruction, not an attribution to the authors.

The inferred rule matches 17 of 18 printed cells. In the generic-prime space capture, modular squaring has numerator 380,314 and grouped denominator 3,608,043. Their ratio is about 10.54 percent, which rounds to 11 percent rather than the printed 10 percent. Classes that individually round to zero make reverse inference nonunique even when a row total agrees.

The special-prime controlled-modular-adder row illustrates a second ambiguity. Selecting the reported controlled modular adder and its inverse gives the printed 39 percent. Adding the separately captured pair of `HandleP` classes contributes another six percentage points and gives 45 percent, yet the source provides no rule explaining their exclusion. The complete inferred class map and both denominators appear in Appendix C.

7 Boundaries of the formal, source, and statistical evidence

The formal record is VQ revision 00ce72179927, with theorem sources under `VQ/Experiments`, paper-facing tests under `Tests`, and the exact theorem index in Appendix A. The retained report identifies the aggregate used for this audit. Its exact locators are:

Report: `paper/schrottenloher-final-audit.md`

Job: `schrottenloher-final-synthesis-aggregate-20260821-1`

The aggregate command `lake --rehash --reconfigure build Tests` completed 4,596 of 4,596 build jobs with status zero under Lean 4.32.0. That result establishes that the named statements and their imports checked at the pinned revision. It does not remove a theorem premise, extend a circuit domain, validate external bytes, or derive an unproved composition.

The source-execution record consists of the directories `schema-v2-baseline` and `schema-v2-full-window` under `paper/schrottenloher-resource-capture`. Both directories retain two manifests and four result files under the same names. Four tracked programs created and tested the records. The exact names are:

Record files:

`manifest-run-1.json`

`manifest-run-2.json`

`special-prime-space-efficient.result.json`

```

special-prime-gate-efficient.result.json
generic-prime-space-efficient.result.json
generic-prime-gate-efficient.result.json
Tools:
tools/schrottenloher-resource-capture.py
tools/test-schrottenloher-resource-capture.py
tools/run-schrottenloher-resource-captures.py
tools/test-run-schrottenloher-resource-captures.py

```

The static source record is `paper/schrottenloher-resource-audit.md`, supplemented by the checked arithmetic modules for Schrottenloher’s Tables 1–3. The 402-round schedule-source and probability-evidence audit is `paper/schrottenloher-402-source-audit.md`. The round-369 value counterexample is in the formal modules and tests indexed in Appendix A. Appendix D identifies retained capture programs, runner, tests, manifests, and payloads by SHA-256 and records manifest hashes for the inspected source trees.

The statistical record comprises exactly the five tracked files named in Section 1. Lean theorems check the listed trace lengths, checkpoints transcribed from the manifest, binomial definitions, a rational test case, and the rational threshold comparison. The million-trial sample, its raw-byte provenance, and the decimal interval enclosures come from external execution under the manifest’s stated environment. The missing raw transcript URI prevents independent retrieval of the exact 32 MB sample from the evidence repository.

The bibliographic record is Schrottenloher v1, the pinned companion and Qarton revisions, the schedule source, the cited binary-Euclid analysis, Babbush et al. v2 and DOI-tagged public bundle, and the Clopper–Pearson paper. The Babbush comparison in this report checks Appendix arithmetic and rounding from their published v2 bounds. It neither inventories nor verifies the Zenodo program, outputs, verification keys, or Groth16 proofs.

Lean’s kernel checks the formal declarations and their proof terms [5]. Those declarations import Mathlib [14] and rely on faithful transcription of the paper’s parameters and source quantities. Captured results additionally rely on CPython, Qarton, installed dependencies, and the instrumentation that records root events and SWAP status. Statistical conclusions rely on the stated sampler, IID premise, transcript checker, and external interval evaluator. The absence of a publication lockfile and the absence of the raw statistical transcript are separate limits: the first prevents exact reconstruction of the authors’ software environment, while the second prevents retrieval of this audit’s sample bytes.

8 Evidence required to close unresolved claims

Table 8: Evidence that would change an unresolved disposition.

Unresolved item	Missing evidence	Conclusion supported by that evidence
The 402-round schedule has the stated tail	A fixed-denominator secp256k1 theorem under the induced multiplier-input law, or a revised deterministic schedule with its final layout and point-addition cost	A valid termination contribution to the paper’s aggregate bound, or a costed replacement for that contribution
Algorithm 1 supplies the reference-law arithmetic inputs	A theorem deriving its arithmetic-call law from the basis-label execution, or event-specific domination bounds for every selected call	Transfer of the conditional component bounds to the generated point-addition computation
Schrottenloher’s Table 1 has the stated complete failure probability	One circuit event space containing termination, shrinking-field validity, truncated comparisons, modular arithmetic, and cleanup, with a union theorem respecting every conditional premise	A checked aggregate bound for the exact paper circuit and input law
Schrottenloher’s Table 1 total-gate estimates follow from source	The included gate types and weights, summation or aggregation rule, and rounding rule	Reproduction or correction of the printed total-gate column from pinned reporter data
Schrottenloher’s Table 3 percentages follow from source	The authors’ class-selection map, inverse-class treatment, overlap policy, and per-class or post-sum rounding order	A unique reconstruction of all 18 cells and resolution of the generic modular-square discrepancy
The resource execution reproduces the publication environment	An exact dependency lock, immutable base-image identifier, and publication command that constructs the intended full lookup table	Re-execution under the authors’ software environment rather than the retained August 2026 environment
Captured widths follow from program syntax	A derivation from Python construction and memory-manager transitions to every dominant root event and peak-liveness value	A syntax-derived width result independent of instrumentation provenance

Unresolved item	Missing evidence	Conclusion supported by that evidence
Schrottenloher’s Table 2 is a whole-circuit result	Generated whole-circuit syntax or separate bounds for the initial lookup, QFTs, measurement, and their composition and liveness	A supported whole-circuit interpretation rather than a 28-addition core formula
Babbush circuit-level comparison	Released circuits with a checkable semantic and resource model, plus bounds for omitted lookup and recycled-QFT terms	Circuit-level checking and an end-to-end whole-circuit resource interpretation
Babbush attested computation	An independent audit of the public program, outputs, verification keys, and Groth16 proofs	Validation of the attested resource-and-test computation, without releasing the circuits or proving universal circuit correctness

Each item in Table 8 concerns a claim or source named in Section 1. A result would change a disposition only if it matches the same circuit model, input law, register layout, and resource convention. A stronger component theorem under a different model would require a checked bridge before it affected the paper-facing conclusion.

9 Conclusion

The checked functional components include the ideal Fourier-state recovery reference result, Algorithm 1’s generated register path and affine-result formula under explicit premises, the binary-Euclidean trace and compressed record, pure reversed Bézout reconstruction, the conditional in-place multiplier, and the exact modular operations in Algorithms 5 and 8 under their stated input and clean-work premises. Separate Algorithm 1 value lemmas cover enabled addition and disabled zero selection under their respective premises, without a composed circuit-to-group theorem. Algorithms 6, 7, and 9–11 have checked circuit action on their named success domains and reference-law probability results for their selected events. These statements distinguish arithmetic value, reversible action, cleanup, and probability rather than treating one as evidence for the others.

The Euclidean findings include a checked residue that needs 405 rounds and a deterministic alternative based on the tight universal full-width bound 511. The resulting 512-round exact-adder schedule has an 855-bit record that does not fit the paper’s 748-position shared allocation. A separate round-369 witness violates the publication shrinking schedule’s trace-domain premise despite an agreeing comparison. Resource executions reproduce the four grouped baseline counts and identify discrepancies in two baseline widths. Static arithmetic over the full-window source establishes the 201,410 surcharge and the two displayed exponent corrections in Schrottenloher’s Table 2. Captured output and Qarton’s recurrence establish the 15-qubit peak-width increment. Under the audit-inferred map for Schrottenloher’s Table 3 and rounding order, generic modular squaring is about 10.54 percent and rounds to 11 percent rather than the printed 10 percent. The authors’ map is absent.

The aggregate failure probability in Schrottenloher’s Table 1 and the whole elliptic-curve discrete-logarithm resource claims remain unresolved. Closing them requires the induced Algorithm 1 call law or event-specific domination, a common event space and conditional union theorem, a supported termination schedule and layout, the missing resource aggregation rules, and stronger provenance for the publication and Babbush computations. Those additions would determine whether the checked component results compose to the paper’s complete circuit.

A Formal evidence index

Every entry in this appendix belongs to VQ revision 00ce72179927. A row summarizes the checked proposition and retains its material premises. The source file contains the full type. The listed test is the paper-facing test module or the test that imports and checks the statement, not an assertion that every theorem in that module has the same scope.

Table 9: Declaration, proposition, premises, and test index.

Item	Exact declaration and source	Proposition checked	Material premises	Test module
§2	<code>ReferenceVerification;</code> <code>secp256k1_reference_verification</code> <code>Tests/ECDLPFourierRecovery/Verified.lean</code>	Emitted ideal reference state, normalized marginal, selected mass, recovery, and reference resources for secp256k1	Semantic level at least 257; valid point; $d < r$; $Q = [d]P$	Same file
§2 reference law	<code>referenceMultiplierLaw_mass;</code> <code>referenceMultiplierLaw_probability</code> <code>VQ/Experiments/SchrottenloherFiniteProbability.lean</code>	Exact point mass and event-cardinality formula for the fixed reference law	Uniform nonzero Euclidean input and independent uniform reduced coefficient	<code>Tests/SchrottenloherFiniteProbability.lean</code>
§2 input law	<code>card_algorithm0BasisLabel;</code> <code>algorithm0BasisLabel_not_pushesForward_reference</code> <code>VQ/Experiments/SchrottenloherPaperInputBridge.lean</code>	The two paper basis-label registers contain 2^{512} labels. No deterministic map pushes their uniform law exactly to the reference law.	Arbitrary candidate map and finite uniform laws	<code>Tests/SchrottenloherPaperInputBridge.lean</code>

Item	Exact declaration and source	Proposition checked	Material premises	Test module
Alg. 1	selector16_point_addition_wellFormed; selector16_point_addition_act; selector16_point_addition_affine; selectedPoint_groupAdd; selectedPoint_algorithm1_true; selectedPoint_algorithm1_false Tests/SchrottenloherAlgorithm1.lean; algorithm1Pre_iff VQ/Experiments/SchrottenloherAlgorithm1Assumptions.lean	The generated path is well formed and yields the stated whole-register affine result. Separate lemmas identify the enabled result with <code>Curve.addPoint</code> , the disabled (0,0) result with the target, and <code>groupAddValue</code> with group addition	Clean work, reduced values, valid shift, unequal x , and positivity for the circuit. <code>GroupValid</code> , unequal x , and recovery noncollision for enabled addition. Representable target and nonzero target x for disabled zero selection	Tests/SchrottenloherAlgorithm1.lean
Alg. 2	compressedTraceCircuit_inputPair; compressedTraceCircuit_record; compressedTraceCircuit_clean VQ/Experiments/SchrottenloherCompressedTraceSemantics.lean	Parameterized circuit produces the mathematical pair and compressed record and clears decision scratch	Positive width; odd left input; both inputs fit the width	Tests/EuclidInPlaceMul.lean
Alg. 2 source	paperScheduleLegal; paperScheduleCircuit_wellFormed; paperScheduleGates_action VQ/Experiments/SchrottenloherArtifactSchedule.lean	Publication shrinking schedule has conditional circuit action, raw/work cleanup, and record-block semantics	ArtifactTraceDomain; odd left; final <code>PaddingSafeAt</code> ; clean raw/work/record	Tests/SchrottenloherArtifactSchedule.lean
Fig. 1	validChoice_card; validChoiceTriple_fits_five_bits; compressedChoiceTriple_lt; compressedChoiceTriple_injective; paperCompressionGates_length; paperCompressionGates_ccx; paperCompressionGates_cx; paperCompression_last_bit_zero; paperCompression_reverse VQ/Experiments/EuclidInPlaceMul.lean; paperCompressionFamily_read_valid_block; paperCompressionFamily_valid_block_lt; paperCompressionFamily_reverse VQ/Experiments/SchrottenloherCompression.lean	Three valid choices fit five bits. The compression map is bounded and injective. The 13-gate compressor has the checked CCX/CNOT counts, clears the sixth bit, reverses valid triples, and supports valid placed-block readout.	Valid choice triple and in-range family block	Tests/EuclidInPlaceMul.lean
Alg. 3	decodeCompressedRecord_compressedTraceRecord; reconstructCompressed_of_terminal; reconstructCompressed_compressedTraceCircuit_of_terminal; reconstructCompressed_two_mul_bits_of_prime; reconstructCompressed_compressedTraceCircuit_two_mul_bits_of_prime VQ/Experiments/SchrottenloherCompressedReconstruction.lean	Pure circuit-record reconstruction decodes the recorded blocks and yields $(0, xy \bmod p)$ after a terminal trace	Odd modulus; values fit; reduced y ; terminal recorded trace	Tests/EuclidInPlaceMul.lean
Alg. 3 generated	circuit_act; circuit_traceInput_act; circuit_two_mul_bits_of_prime VQ/Experiments/SchrottenloherPseudoMersenneRawChoiceSchedule.lean	Generated reverse schedule has the selected coefficient-step action and prime-width multiplication corollary	Schedule bounds, clean state, and per-step pseudo-Mersenne success premises	Tests/SchrottenloherPseudoMersenneRawChoiceSchedule.lean
Alg. 4	circuit_multiplies; circuit_multiplies_of_referenceMultiplierSuccessConditions VQ/Experiments/SchrottenloherPseudoMersenneSecp256k1.lean	Checked 402-round circuit maps (x, y) to $(x, xy \bmod p)$ and restores its trace state	Input bounds; terminal 402-step trace; reconstruction and approximate-success premises	Tests/SchrottenloherPseudoMersenneSecp256k1.lean
Alg. 4 corrected	correctedCircuit_act VQ/Experiments/SchrottenloherDeterministicCorrection.lean	Exact-adder 512-round circuit multiplies and cleans for the corrected layout	Nonzero reduced input; reduced target; clean registers	Tests/SchrottenloherDeterministicCorrection.lean
Alg. 4 termination	euclidStep_product_double_le; euclidTrace_product_bound; euclidStep_bitLengthSum_lt; euclidTrace_steps_add_final_bitLengthSum_le; traceSucceeds_two_mul_bits_sub_one_of_coprime; traceSucceeds_two_mul_bits_sub_one_of_prime VQ/Experiments/EuclidInPlaceMul.lean; all_nonzero_inputs_succeed_511; termination_bound_is_tight Tests/SchrottenloherDeterministicCorrection.lean	Product shrink is a separate invariant. Strict bit-length-sum descent gives the 511-round universal bound, and 2^{55} proves tightness	Positive bit width; coprime or prime specialization; nonzero reduced input	Tests/SchrottenloherDeterministicCorrection.lean
Alg. 4 shrinking	domainFailureState_step_mismatch_369 Tests/SchrottenloherArtifactTrace.lean; one_div_p_sub_one_le_valueFailureProbability VQ/Experiments/SchrottenloherArtifactTraceFailure.lean	One publication-schedule input has divergent full and clipped successors; its point mass lower-bounds uniform-nonzero failure	Named witness; publication shrinking model; uniform nonzero law for probability	Tests/SchrottenloherArtifactTrace.lean; Tests/SchrottenloherArtifactTraceFailure.lean
Alg. 5	paperLocalGates_act; circuit_refines_algorithm5; circuit_doublesMod; secp256k1_program_width; secp256k1_program_toffoliCount; secp256k1_program_cnotCount VQ/Experiments/SchrottenloherAlgorithm5.lean	Five-stage circuit refines the pseudocode and doubles modulo an odd modulus. The isolated secp256k1 program has width 517, 3,340 Toffoli gates, and 1,542 CNOT gates.	Odd modulus, fitting width, reduced input, and clean work	Tests/SchrottenloherAlgorithm5.lean
Alg. 6	circuit_act VQ/Experiments/SchrottenloherEfficientDoubleBoundary.lean; algorithm6FailureProbability_le VQ/Experiments/SchrottenloherGenericPrimeAlgorithm6Probability.lean; algorithm6Secp256k1FailureBridge_event_eq VQ/Experiments/SchrottenloherGenericPrimeAlgorithm6Secp256k1Bridge.lean	Success-domain doubling action; generic high-part-gap bound; equality of generic and fixed circuit events	Reduced input; efficient-double success; generic-prime parameters; reference law for probability	Tests/SchrottenloherEfficientDouble.lean; Tests/SchrottenloherGenericPrimeAlgorithm6Secp256k1Bridge.lean
Alg. 7	circuit_act VQ/Experiments/SchrottenloherPseudoMersenneDoubleBoundary.lean; referenceMultiplierFirstDoubleFailureProbability_lt_two_pow_neg_40; referenceMultiplierFirstDoubleFailureProbability_sum_lt_two_pow_neg_31 VQ/Experiments/SchrottenloherPseudoMersenneFailure.lean	Success-domain doubling action and per-index/402-index-sum bounds for the combined event	Reduced input; pseudo-Mersenne success; reference law	Tests/SchrottenloherPseudoMersenneDouble.lean; Tests/SchrottenloherPseudoMersenneSecp256k1.lean
Alg. 8	paperLocalGates_controlled_addsMod VQ/Experiments/SchrottenloherAlgorithm8Semantics.lean; canonicalCircuit_refines_algorithm8; canonicalCircuit_act; secp256k1_program_width; secp256k1_program_toffoliCount VQ/Experiments/SchrottenloherAlgorithm8Resources.lean	Five-stage circuit and canonical resource circuit produce the exact controlled modular sum and clean work. The isolated secp256k1 program has width 773 and 10,767 Toffoli gates.	Positive fitting modulus, reduced source and target, clean work, and selected exact adder	Tests/SchrottenloherAlgorithm8.lean
Alg. 9	circuit_act_of_success VQ/Experiments/SchrottenloherEfficientControlledAddSemantics.lean; referenceMultiplierFirstEfficientAdditionFailureProbability_lt_two_pow_neg_39 VQ/Experiments/SchrottenloherGenericPrimeAlgorithm9Secp256k1Bridge.lean	Success-domain generic controlled addition and per-index combined-event bound	Reduced values; efficient-add success; reference law	Tests/SchrottenloherEfficientControlledAdd.lean; Tests/SchrottenloherGenericPrimeAlgorithm9Secp256k1Bridge.lean

Item	Exact declaration and source	Proposition checked	Material premises	Test module
Alg. 10	circuit_act_of_success VQ/Experiments/SchrottenloherPseudoMersenneControlledAddSemantics.lean; referenceMultiplierFirstAdditionArithmeticAfterDoubleProbability_lt_two_pow_neg_40 VQ/Experiments/SchrottenloherPseudoMersenneFailureProbabilityRational.lean; referenceMultiplierFirstAlgorithm10CleanupAfterPriorSuccessProbability_lt_two_pow_neg_40_via_genericPrime VQ/Experiments/SchrottenloherGenericPrimeAlgorithm10Secp256k1Bridge.lean	Success-domain pseudo-Mersenne controlled addition and two conditional per-index bounds	Successful doubling or named prior success; reference law	Tests/SchrottenloherPseudoMersenneControlledAdd.lean; Tests/SchrottenloherGenericPrimeAlgorithm10Secp256k1Bridge.lean
Alg. 11	circuit_act_of_success VQ/Experiments/SchrottenloherPseudoMersenneControlledAddEqualitySemantics.lean; algorithm11Secp256k1ClosedControlSplitFailureProbability_lt_two_pow_neg_38; algorithm11Secp256k1AllIndicesFailureProbability_lt_two_pow_neg_29 VQ/Experiments/SchrottenloherGenericPrimeAlgorithm11Secp256k1EventBridge.lean no_disabledEqualityPrefixFiberData VQ/Experiments/SchrottenloherGenericPrimeAlgorithm11CoefficientFiberCounterexample.lean	Equality-aware success-domain action; control-split per-index and 402-index-union bounds	Reduced values; equality-aware success; prioritized classes; reference law	Tests/SchrottenloherPseudoMersenneControlledAddEquality.lean; Tests/SchrottenloherGenericPrimeAlgorithm11Secp256k1EventBridge.lean
Alg. 11 audit interface	no_disabledEqualityPrefixFiberData VQ/Experiments/SchrottenloherGenericPrimeAlgorithm11CoefficientFiberCounterexample.lean	At $p = 13$, the proposed control-agnostic disabled-equality prefix-fiber data do not exist	The attempted audit interface's fixed small-prime parameters	Tests/SchrottenloherGenericPrimeAlgorithm11CoefficientFiberCounterexample.lean
Schrottenloher's Table 1 baseline	sourcePointCosts VQ/Experiments/SchrottenloherTable1StaticSourceEvidence.lean; capturedGroupedTotals_matchStaticSource VQ/Experiments/SchrottenloherTable1CapturedEvidence.lean; baselineCertificates_applyRecurrence; baselineCertificates_matchCapturedRows VQ/Experiments/SchrottenloherTable1PeakWidthEvidence.lean	Static point-cost equations equal captured grouped totals. Recorded baseline root events satisfy the Qarton recurrence and captured rows.	Transcribed pinned-source constants, captured integers, and recorded events	Tests/SchrottenloherTable1StaticSourceEvidence.lean; Tests/SchrottenloherTable1CapturedEvidence.lean; Tests/SchrottenloherTable1PeakWidthEvidence.lean
Schrottenloher's Table 1 full window	sourceFullWindowPointCosts; sourceWindowGroupedIncrement_eq; sourceWindowIncrement_exceedsPaperBy VQ/Experiments/SchrottenloherTable1StaticSourceEvidence.lean; capturedExecutionRecords_groupedTotals_matchStaticFullWindowCosts; capturedExecutionRecords_fullWindowWidths_eq_baselinePlusFifteen; capturedExecutionRecords_rootPeakRecurrence_withSeventeenOutsideQubits VQ/Experiments/SchrottenloherTable1FullWindowCapturedEvidence.lean	Static formulas give the four full-window totals and increment 201,410, exceeding the paper surcharge by 4,802. Widths are baseline plus 15. Root events satisfy the recurrence with 17 qubits outside the child.	Captured schema-v2 full-window records and static pinned-source arithmetic	Tests/SchrottenloherTable1StaticSourceEvidence.lean; Tests/SchrottenloherTable1FullWindowCapturedEvidence.lean
Schrottenloher's Table 2 source	windowSchedule_partition VQ/Experiments/SchrottenloherTable2ScheduleEvidence.lean; sourceCounts_eq; sourceSpecialSpace_roundsTo2611; sourceSpecialGate_roundsTo2579; sourceGenericSpace_roundsTo2667; sourceGenericGate_roundsTo2633; sourceSpecialGate_doesNotRoundTo2578; sourceGenericSpace_doesNotRoundTo2666 VQ/Experiments/SchrottenloherTable2StaticSourceEvidence.lean	The schedule leaves 28 additions. Exact full-source totals round to 26.11, 25.79, 26.67, and 26.33. The two disputed entries do not round to the printed values.	$n = 256$, $w = 16$, and exact full-window source counts	Tests/SchrottenloherTable2ScheduleEvidence.lean; Tests/SchrottenloherTable2StaticSourceEvidence.lean
Schrottenloher's Table 2 Babbush	appendixACoreGateBounds_eq; appendixAQubitBounds_eq; lowQubitPointGateBound_roundsTo2136; lowGatePointGateBound_roundsTo2100; lowQubitAppendixACoreGateBound_roundsTo2627; lowGateAppendixACoreGateBound_roundsTo2594 VQ/Experiments/SchrottenloherTable2BabbushEvidence.lean	Babbush v2 point-addition bounds and Schrottenloher's core formulas equal the stated gate and qubit integers. The gate values round to 21.36, 21.00, 26.27, and 25.94.	Transcribed v2 bounds and formulas. No circuit or deposit verification.	Tests/SchrottenloherTable2BabbushEvidence.lean
Schrottenloher's Table 3	specialReconstruction_matchesPaper; genericReconstruction_matchesPaperExceptModularSquare; genericReconstruction_modularSquare_eq VQ/Experiments/SchrottenloherTable3CapturedEvidence.lean	Audit-inferred map matches all special cells and eight generic cells. The generic square reconstructs as 11.	Captured denominators/numerators and audit-attributed class/rounding map	Tests/SchrottenloherTable3CapturedEvidence.lean
Statistics: binomial definitions	binomialPointMass_eq_measure; binomialLowerTail_eq_sum_measure; binomialLowerTail_eq_measure; oneSidedPValue_eq_measure VQ/Experiments/SchrottenloherBinomialAudit.lean	The audit's finite sums agree with Mathlib's binomial measure and one-sided lower-tail p-value.	Natural trial and failure counts, with probability in $[0, 1]$ where required	Tests/SchrottenloherBinomialAudit.lean
Statistics: zero failures	companionZeroFailureWeight_eq; companionZeroFailureWeight_gt_one_third; companionZeroFailureWeight_lt_half Tests/SchrottenloherSampling.lean	At failure rate $1/10000$, the 10,000-trial zero-failure weight equals $(9999/10000)^{10000}$ and lies strictly between one third and one half.	IID Bernoulli model expressed by the imported repetition measure	Same file
Statistics: checkpoints	hasExactLength; #7 guard checks Tests/SchrottenloherBinomialAuditCheckpoints.lean	All 19 recorded failures and all 18 manifest checkpoints have the transcribed exact Euclidean length.	Fixed secp256k1 trace and transcribed values	Same file

The $p = 13$ row records an audit-interface correction. It disproves the audit's first attempt to use one control-agnostic coefficient-fiber object for Algorithm 11. It does not contradict Schrottenloher's algorithm or its stated probability estimate. The final checked bound instead uses the control-split interface indexed above.

B Counterexamples and deterministic alternative

B.1 Trace longer than 402 rounds

The checked nonzero secp256k1 residue

$$x = 52c7d5143f4c806315b9f9f28fbd6c071873e6c89587b06f9e9f8ee1d7e0cad2 \quad (2)$$

leaves the full-width Euclidean state $(1, 7)$ after 402 transitions. The state is nonterminal at that point and reaches $(1, 0)$ at transition 405. The checked declarations are `longTraceInput_after_402`, `longTraceInput_fails_402`, and `longTraceInput_succeeds_405` in `Tests/SchrottenloherPseudoMersenneSecp256k1.lean`.

This witness establishes that the fixed 402-round specialization is not a total termination schedule. Under the uniform-nonzero law, the displayed input contributes probability $1/(p-1)$. The witness does not settle Schrottenloher’s Table 1 aggregate numerical bound, which still requires a denominator count or another valid distributional argument together with the other row-level events and their dependence structure.

B.2 Tight bound and 512-round layout

The declarations `euclidStep_product_double_le` and `euclidTrace_product_bound` prove a product-shrink invariant, but that invariant does not yield the sharp 511-round result. The exact proof uses `euclidStep_bitLengthSum_lt` and `euclidTrace_steps_add_final_bitLengthSum_le`: every live step strictly reduces $\text{bitLength}(u) + \text{bitLength}(v)$, and the terminal pair retains bit-length sum one. The coprime and prime corollaries `traceSucceeds_two_mul_bits_sub_one_of_coprime` and `traceSucceeds_two_mul_bits_sub_one_of_prime` specialize this descent to the $2n-1 = 511$ bound. The input $x = 2^{255}$ is at $(1, 1)$ after 510 rounds and first reaches the terminal pair at round 511, as checked by `powTwo255_after_510`, `powTwo255_fails_510`, and `powTwo255_succeeds_511`. Thus 511 is the exact universal bound for this full-width transition, not an empirical cutoff.

The corrected reversible circuit executes 512 decisions so that the terminal transition and fixed padding have a uniform schedule. Its exact-adder theorem `correctedCircuit_act` covers every nonzero reduced input and reduced target with the work and record registers initially clean. The theorem proves the modular product and endpoint cleanup without an approximate-arithmetic success assumption.

Storage prevents this alternative from inheriting the paper’s resource row. The compressor needs

$$\left\lceil \frac{512}{3} \right\rceil = 171 \quad \text{blocks and} \quad 171 \cdot 5 = 855 \quad \text{record bits.}$$

The two reported 374-bit shared fields provide $2 \cdot 374 = 748$ positions, so $855 > 748$. The declarations `deterministicRecord_exceeds_paperFields` and `no_paperFieldWidth_scheduleLegal` check the storage incompatibility. No theorem in the pinned evidence gives a whole point-addition cost after replacing the paper schedule with this layout.

B.3 Round-369 shrinking-field witness

The publication shrinking schedule has the checked input

$$x = \text{ac2ad9731956a0318967f615ff0085ca6c38d615de6ffcf3e390f25d79000854}. \quad (3)$$

After 369 rounds its full state is $(6033964351, 44029)$. The effective comparison of width $\min(\text{currentWidth}, 77)$ agrees with the full comparison and selects the same ordered odd branch. The relevant checked facts are `domainFailureState_after_369`, `domainFailureState_padding_fails_369`, and `domainFailureState_comparison_agrees_369` in `Tests/SchrottenloherArtifactTrace.lean`.

The next successors differ despite the agreeing branch:

$$\text{full} = (44029, 3016960161), \quad \text{clipped} = (44029, 869476513).$$

The declarations `domainFailureState_algorithm2Step_369`, `domainFailureState_artifactStep_369`, and `domainFailureState_step_mismatch_369` check those values and their inequality. The high bits are lost from a live operand before the following update, so comparison correctness alone cannot discharge `ArtifactTraceDomain`.

Under the uniform law on $\{1, \dots, p-1\}$, one failing input gives a probability contribution exactly $1/(p-1)$. The theorem `one_div_p_sub_one_le_valueFailureProbability` in `VQ/Experiments/SchrottenloherArtifactTraceFailure.lean` applies that one-point argument to the shrinking trace model. It neither estimates the number of other shrinking failures nor combines this event with termination, comparison, arithmetic, or cleanup failures.

C Resource arithmetic and class map

C.1 Captured reporter output

Table 10 transcribes the eight canonical schema-v2 result JSON files in paper row order. Width is the captured `Circuit.nbr_qubits()` output, while Bits is the captured `Circuit.nbr_bits()` output. Raw CCZ is zero in every row and is omitted from the table. Adding raw CCX and forward AND gives the displayed grouped total.

Measured uncompute is the count of zero-weight measured-AND uncomputations under the grouped convention, whereas basic sum adds every entry returned by `basic_gate_counts()`.

Table 10: Raw baseline and full-window captured reporter values.

Mode	Width	Bits	Raw CCX	Fwd. AND	Meas. uncompute	Grouped	Basic sum
Baseline special space	1,192	1,541	1,740,787	644,730	526,144	2,385,517	19,929,384
Baseline special gate	1,445	1,541	502,635	1,362,886	1,360,076	1,865,521	31,170,396
Baseline generic space	1,191	1,701	2,430,771	1,177,272	698,494	3,608,043	35,852,486
Baseline generic gate	1,446	1,541	1,325,731	1,477,788	1,474,978	2,803,519	37,862,666
Full special space	1,207	1,556	1,742,547	844,380	725,794	2,586,927	66,002,343
Full special gate	1,460	1,556	504,395	1,562,536	1,559,726	2,066,931	77,243,355
Full generic space	1,206	1,716	2,432,531	1,376,922	898,144	3,809,453	81,925,445
Full generic gate	1,461	1,556	1,327,491	1,677,438	1,674,628	3,004,929	83,935,625

The exact baseline grouped exponents, rounded to the nearest hundredth in the paper’s order, are 21.19, 20.83, 21.78, and 21.42. These values reproduce the four displayed baseline non-Clifford entries. The reproduction is a static arithmetic and captured-output result under the pinned cost convention, not a proof of the total-gate or failure-probability columns.

C.2 Static schedule-source arithmetic

The static source evidence transcribes the publication schedule as 402 rounds with padding 37, compressed-record width 670, coordinate width 374, and remaining padding 78. It also checks adder budget 183, active-width sum 60,124, and retained-comparison-width sum 27,775. These quantities come from pinned schedule-source formulas and exact Lean arithmetic rather than from the point-addition executions.

The same source evidence derives `ToBitVector` grouped costs 219,735 in space mode and 212,569 in gate mode. Together with the other transcribed component formulas, those values produce all four baseline and full-window grouped totals. The baseline totals are checked by `SchrottenloherTable1StaticSourceEvidence.sourcePointCosts`. The full-window totals and surcharge are checked by `sourceFullWindowPointCosts`, `sourceWindowGroupedIncrement_eq`, and `sourceWindowIncrement_exceedsPaperBy` in the same module. These declarations do not derive Python liveness or Schrottenloher’s Table 1 total-gate column.

C.3 Root events and SWAP observation

The root-event tuple below is ordered as $(oldPeak, newCurrent, childPeak, oldCurrent, previousInside)$. Applying Equation (1) produces the final column. The child peaks are unchanged between baseline and full-window runs. The root’s live-current increment changes the result.

Table 11: Captured root-event tuples and recurrence outputs.

Mode	Event tuple	Peak
Baseline special space	(1031, 514, 1190, 514, 512)	1,192
Baseline special gate	(1031, 514, 1443, 514, 512)	1,445
Baseline generic space	(1031, 514, 1189, 514, 512)	1,191
Baseline generic gate	(1031, 514, 1444, 514, 512)	1,446
Full special space	(1057, 529, 1190, 529, 512)	1,207
Full special gate	(1057, 529, 1443, 529, 512)	1,460
Full generic space	(1057, 529, 1189, 529, 512)	1,206
Full generic gate	(1057, 529, 1444, 529, 512)	1,461

Qarton’s asymmetric quantum/classical SWAP update is in `qarton/circuit/memory_manager.py`, lines 529–542, at revision 7506fab2147 [6]. When operand zero is quantum and operand one is classical, the affected branch removes operand zero from the quantum set and adds operand zero again rather than adding operand one. Instrumentation recorded zero reached `qc` cases, zero faulty-branch cases, and zero incorrect transitions in all eight canonical results, so this dormant defect does not explain their widths.

C.4 Surcharge and Schrottenloher’s Table 2 calculations

The full-window grouped increment expands directly from the pinned formulas:

$$\begin{aligned}
3(2^{16} - 2) &= 196,602 && \text{forward lookup,} \\
3(1528) &= 4,584 && \text{charged gates in measured unlookup,} \\
4(56) &= 224 && \text{nonzero tests,} \\
196,602 + 4,584 + 224 &= 201,410 && \text{total.}
\end{aligned}$$

The paper’s $3 \cdot 2^{16} = 196,608$ differs by 4,802. The address grows from one bit to 16 bits, which agrees with the captured width increment $16 - 1 = 15$.

Table 12: Exact core arithmetic for Schrottenloher’s Table 2.

Mode	Paper rounded $\log_2$	28(baseline + 196,608)	Its $\log_2$	Full total	Full $\log_2$
Special space	26.11	72,299,500	26.11	72,433,956	26.11
Special gate	25.78	57,739,612	25.78	57,874,068	25.79
Generic space	26.66	106,530,228	26.67	106,664,684	26.67
Generic gate	26.33	84,003,556	26.32	84,138,012	26.33

The second numeric column equals 28 times the exact baseline grouped count plus the paper’s surcharge inside the parentheses. The full total is 28 times the exact full-window grouped count. Both formulas omit the initial direct lookup, and neither includes QFT or measurement resources.

C.5 Babbush substitutions

The two checked comparison substitutions are

$$\begin{aligned}
28(2,700,000 + 3 \cdot 65,536) &= 81,105,024, & 1,175 + 16 &= 1,191, \\
28(2,100,000 + 3 \cdot 65,536) &= 64,305,024, & 1,425 + 16 &= 1,441.
\end{aligned}$$

Rounding base-two logarithms gives point-addition entries 21.36 and 21.00 and core entries 26.27 and 25.94. These calculations transcribe published Babbush v2 bounds and Schrottenloher’s composition. They do not check the withheld circuits or the public Zenodo computation.

C.6 Complete inferred map for Schrottenloher’s Table 3

The special and generic denominators are 2,385,517 and 3,608,043. Each cell in Table 13 lists selected class numerators followed by the integer percentage obtained after Qarton’s two-decimal per-class rounding and summation. Names ending in `_inv` are separate captured recursive classes.

Table 13: Audit-inferred recursive-class map for Schrottenloher’s Table 3.

Component	Special-prime selection: numerators $\rightarrow$ result	Generic-prime selection: numerators $\rightarrow$ result
Modular square	<code>ControlledSpecialPrimeModularSquareAdd_inv</code> : 207,964 $\rightarrow$ 9	<code>ControlledEfficientModularSquareAdd_inv</code> : 380,314 $\rightarrow$ 11 (paper 10)
In-place multiplier and inverse	<code>IPModMul</code> , <code>IPModMul_inv</code> : 1,080,986 each $\rightarrow$ 45 + 45 = 90	Same names: 1,606,074 each $\rightarrow$ 45 + 45 = 90
Bézout reconstruction	<code>ApplyBitVector</code> , <code>ApplyBitVector_inv</code> : 641,516 each $\rightarrow$ 27 + 27 = 54	Same names: 1,166,604 each $\rightarrow$ 32 + 32 = 64
GCD construction	<code>ToBitVector</code> , <code>ToBitVector_inv</code> : 439,470 each $\rightarrow$ 18 + 18 = 36	Same names: 439,470 each $\rightarrow$ 12 + 12 = 24
Controlled swap	<code>ControlledSwap</code> : 446,320 $\rightarrow$ 19	Same name: 446,320 $\rightarrow$ 12
Hybrid adder	<code>ControlledHybridAdder</code> , <code>ControlledHybridAdder_inv</code> : 254,828 each $\rightarrow$ 11 + 11 = 22	Same names: 254,828 each $\rightarrow$ 7 + 7 = 14
Controlled modular adder	<code>SpecialPrimeControlledModularAdder</code> : 375,584; inverse 541,216 $\rightarrow$ 16 + 23 = 39	<code>EfficientControlledModularAdder</code> : 699,078; inverse 922,310 $\rightarrow$ 19 + 26 = 45
Modular double	<code>SpecialPrimeModularDouble</code> , inverse: 107,340 each $\rightarrow$ 4 + 4 = 8	<code>EfficientModularDouble</code> , inverse: 434,859 each $\rightarrow$ 12 + 12 = 24
Gidney constant adders	<code>GidneyConstantAdder</code> : 3,835; <code>ControlledGidneyConstantAdder</code> : 350,304 $\rightarrow$ 0 + 15 = 15	Same names: 3,835; 1,237,920 $\rightarrow$ 0 + 34 = 34

The generic modular-square ratio is $380314/3608043 = 10.540728 \dots$ percent. The exact boundary calculation $200(380314) - 21(3608043) = 293897 > 0$ places it above 10.5 percent, so the inferred rule yields 11. All other inferred cells equal the paper’s values, giving 17 agreements among 18 cells.

`SpecialPrimeHandlePControlledModularAdder` and its inverse each have numerator 68,550 and rounded contribution 3 percent. Adding both changes the total for the controlled modular adder from 39 to 45. This alternative demonstrates why the missing authors’ class map prevents a unique source reconstruction.

D Statistical and execution provenance

D.1 Statistical inference and run identity

The statistical execution used manifest repository_revision a72e2e28aa5da256e8b0015288e6cdce5aebe6d0, which differs from the formal-evidence revision 00ce72179927. Its manifest records CPython 3.13.5 on aarch64 Linux 6.12.57+deb13 with glibc 2.41, sampling through `os.getrandom` without flags from 19:35:07 to 19:35:26 UTC on 18 August 2026. It stopped after 1,000,000 accepted candidates, rejected zero, recorded 19 failures, maximum length 415, 361,842,246 total Euclidean steps, and a 32,000,000-byte raw transcript.

For the composite null $q \geq 1/10100$ under the IID-uniform accepted-residue model, the one-sided exact-binomial p-value is enclosed by

$$\begin{aligned} & 8.3798656826905476271985335220787562621312046249779483847264991343122804735246240 \times 10^{-23} \\ \leq P \leq & 8.3798656826905476271985335220787562621312046249779483847264991343122804736932623 \times 10^{-23}. \end{aligned}$$

The one-sided 95-percent Clopper–Pearson endpoint lies between

$$\frac{35340977961419490592736597}{1267650600228229401496703205376} \quad \text{and} \quad \frac{17670488980709745296368299}{633825300114114700748351602688},$$

whose decimal enclosures are

$$\begin{aligned} & 0.0000278791158660412077778946104658, \\ & 0.0000278791158660412077778946112548. \end{aligned}$$

These exact strings come from `schrottenloher-tail-statistical-audit-inference.json`. Section 5 rounds the endpoint to the precision supported by the discussion.

The declarations `binomialPointMass_eq_measure`, `binomialLowerTail_eq_sum_measure`, `binomialLowerTail_eq_measure`, and `oneSidedPValue_eq_measure` in `VQ/Experiments/SchrottenloherBinomialAudit.lean` identify the finite sums with Mathlib’s binomial measure. The test `Tests/SchrottenloherBinomialAudit.lean` checks the exact 5/16 lower tail for one failure in four fair trials and the integer threshold $2^{133} < 10100^{10}$. The file `Tests/SchrottenloherBinomialAuditCheckpoints.lean` defines `hasExactLength` and checks all 19 failures and all 18 manifest checkpoints against their exact Euclidean lengths. For the paper’s 10,000-test observation, `companionZeroFailureWeight_eq`, `companionZeroFailureWeight_gt_one_third`, and `companionZeroFailureWeight_lt_half` in `Tests/SchrottenloherSampling.lean` check the exact zero-failure weight and its strict one-third and one-half bounds at failure rate 1/10000.

Table 14: Tracked statistical files and identities.

Record	SHA-256 or identity
Protocol specification file	277dfa9340593a7ae6520c1e356c4d898d24e70ff39425d5915566004af3651
Result document	893d7530f489c61f316295988b971deccb6c16720025996834dbe3d1112b0123
Manifest file	f51f7ead862ced632be24553ae304863e5cb4063010e9fc53ba9202001bb86e6
Inference file	9cc187d00b448d9a1ad3731c6cbb0b5f7c99f9769901549003b29c0780c19662
Failures file	20bdf793f7ae9b9c0820b7a08d09816da8c198ba3d0c07debf4991d524c9fd07
Protocol hash recorded by manifest (bytes and path not retained)	18fc4f3654092393391caac5677ef446275fcbe723ad6347a357c662ce74786c
Audit tool	cf7dcc351195a6f0f2b7a2827c77ccea5092bceaadf5e838d547c337ee0f4b9a
Transcript checker	08d123afbc6c07de80fbb3dcee6e1ef7ae3feb649fd9a84820c9ca8b28568b7
Raw transcript	fcdbdf7beb91a20289b2debb50479fd9bc0a446db231d9099737b08d7e95ecefef

The five tracked statistical files are listed in Section 1 and stored under `paper/`. The raw path was `tmp/schrottenloher-tail-iid-20260818/candidates.bin`. It has no durable URI. Its hash authenticates bytes available during checking but does not prove the entropy source or IID premise. The retained protocol and manifest have no independent timestamp that establishes preregistration.

D.2 Failures and fixed checkpoints

Table 15 is the complete tracked failure record. Accepted index counts accepted residues rather than raw draw attempts. Each hex value has a Lean-checked trace length, and the transcript checker matched its index and value to the raw bytes available during the audit.

Table 15: All accepted residues with trace length above 402.

Index	Input hexadecimal	Steps
1,603	f330b51486064c75c9dfbb33799267f2e1d04c75a9084dd6e018ffa7155fa6bc	403
72,384	ee9761014133d4630dba7f33719415f0b2ff0a01ea5947b10457aa5690ccab3a	409
112,203	ca514b4002c0281524f4f88ef7b49fe9cd7071f995eb2e21a15fc4c5f274c39	405

Index	Input hexadecimal	Steps
119,539	fad382c49e2472f0b694d87c07489b40a7df44b147d1629ab707419be5e0670a	406
184,962	e9db47ba7c4973054370c787977a4ee2f6f12c63c331ca882d4391d35f3a97	408
316,336	d1ee090ef28dac9dc60153577259bfb1c04d15e12feed9ce0b0904fcec638	403
445,667	7b8f194b00c4f9db150547d2f080aa47402034ecbdec5530b853dd700807ed68	405
534,197	17c113033b4c2d13a6432c37beb305cb078ca9cda64499c8c250c7e477a3364	409
539,833	7a2ad9ebf835d793a1b13e042ca3f42dcd32f99473d4d2a9bebe3dc2175a737c	415
540,966	718a1a8ec060dd270ae3bbcb302d4f4e3030ad456c6176f0cbda55846a59b10	404
548,934	a898e0782e13a26a0eda6eae8dbf807f51aaf4d6d806c6d4101881da3cd49f4c6	403
575,265	e8b785e63fa70280211f0aa9934bb38ef2f777b2d0ea1e75b2565adfc2dc8f2a	406
610,405	916f733cb95dedf1ff57ea987a86d344b5e7088fc6e9a3a78285564369ecb49e	407
687,025	c8f61d0b82d6e7ac8725a8bf9499b01a006fe6c474974d6885c664a4479878b	406
784,238	4bda31880c9a4e815676720d40866c46e4dd6ab6506f8f3cef3552d8f2c29e	403
791,939	3a58052878f7b29003177a2182b1349f308443e689f9ca471bd2979d5f469c9e	405
828,678	1c730e2ad3924563c779ef3a600d81f8a628e0090314bce405d8a33271f52459	406
888,266	f913cdeb0d99d78ebe28fcc7e18a1da83de95542776cf7389d4b3a4891b1a820	407
973,930	818b1770d542c5f1fabcc64f003966e417e829bfde3be3dd3d9abad39838f4	404

The manifest embeds the 18 checkpoints in Table 16. The first eight check the start of the accepted sequence, and the remaining ten check each 100,000th accepted position. They are provenance anchors rather than a second statistical sample.

Table 16: Fixed checkpoints embedded in the statistical manifest.

Index	Input hexadecimal	Steps
1	b4edc433e7dd99f3bdf3797d236c44d254b4c477ab94ae605ea94f6442dd2882	355
2	bd757c0f2b6b8317737595dd8e69e48c23872c2d9aa54fbd5da82a109c505fc	371
3	10c048a90966a2eda14aa0cb8e628bed5b9cf03936057d63b56ef5ce49573ccef	366
4	6d8a7a785c47770d63dc38c1494e177089f3f81753d557aab51856732c064d9d	369
5	e943ad80247b598d102593a20cf129c1cd63aebb4959c0d61d5f825da341067	358
6	df88cc60335bd431c4508271a595a5e40b204d6298ff0b837e2c1dcfa9a273c	356
7	41ec8176ca882cc04208544f71d8b3c0fba0269e0ada338f6c7f4082a79c1c54	374
8	8890da9ee56c277f8b4bc0c1ed84f9aceb700eb2b0ba812684956b95a90ded2	361
100,000	4f75b606170514bcae4f372cb574e318feeea08e1dfa02b0d45684d9ea9c3a14	377
200,000	48eba7f08c111fa69a6a29b7bc458150191b3710a7dbf6716538152c26b4efd	365
300,000	4ae8426fb73aa5b964f0e1431ed82f2aa64feca36782887eb99c01746683f6	364
400,000	897bec522c69f17b47c45df57baea5fe116c0ec290a151db0ccdc5e9784bb	370
500,000	61faf31eb7e2e18345242c06c222e5fab29e990ff83ab56c136b74d8d5eb2830	366
600,000	cd9a865712d36b5079627d1ce9a83f0a2050b184de0afd7f6fb4dce455859497	364
700,000	330cbbe983275aee6d8016ac0950800443c9b46a5b37cfa5a56558eb4fa764c44	354
800,000	ld164ec3e7401880664442839822f4b709d293240d8f739a1ac4b7ad4aa1704f7	361
900,000	9fe5328da3f2bd1d73405f53442d0b682b2c1f12513c820bc3bca549e8b16da3	354
1,000,000	0d3d6d38da8672336377e868ad325fd226e847d68d4b89cec5bf791e2e3688f9	365

D.3 Resource capture identities

The four resource suites ran on 21 August 2026 UTC, corresponding to the evening of 20 August in the audit’s local time. They used uv 0.10.2, CPython 3.12.12 invoked with `-I -B`, Qarton 0.2.1, companion revision 3b60d5050b18, and Qarton revision 7506faba2147. All mode processes returned status zero with empty standard error, and the two results for each record class were byte-identical.

Table 17: Resource manifest, tool, and source identities.

Object	SHA-256
Baseline manifest 1	3a383fe577ffa7eb3f292438a14c453364d5bb651531db3ca1901feba9e8c198
Baseline manifest 2	04c358b7d21fe8c77d99047ffc50015f3050552828c41416c27689368734c02f
Full-window manifest 1	6dae9923110b6ec084c7bc4c62337f240a8bf724cfb35128748f9d994c28e2c
Full-window manifest 2	0b6186d69d60f93e9ede5fcd063d21e7d0ea1af9fc77f98d36a11dba25c55cd
Capture program	03eab7cfce28bd3af68f060b3e4d0f35a65725a87f33a0d6ac5cf821ead5a3aa
Capture test	820dd423b0406d60a5ed6cca417ce36e7e1df9f0212a333157acd4f5770f6594
Four-mode runner	bed00916e1c18d3e8480587ecee6ab3685fd35525207e6a2d50d5ae30c53c981
Runner test	67aa46d984621eadb49194a13dab5531ce8d23b18490e1d8ad81466fcc30a8d3
Full table digest	85d3a20ffa586ffcc144d125f0a662246c3ba82afeec84a9905e3ec25ddde380
Companion source archive	74fb7bb4c691b49317227c9960393c39c3dfa7365345e98bf24813a8bd6b40ca
Qarton source archive	708e0ad1b0bd1f47019fc4abee78f82ad0e8622a6730dd0cd7cac8934bf892bf
Bundled publication PDF	1f80b0757839aa925f791e3bfe37462b29685fffd8e355d347e5176e4ec9b1d
Companion 15-file Python tree	d303ee4ec339d77c019ae3e454ffbf6f408294d15ad61422ed05673561d863a7d
Qarton 297-file Python tree	a48164ad760c5744b9d0b22c26d96d586806bcd6f794b12f6cb4588c4323b82
Companion build_circuit.py	a12d4f57df9dea777f1ae718bea48aa634e91685742f108df2c64c1b3bcf14f3
Installer	feef1a6b1e055dff5e52d23441c6a50e231ee941acac23943aa8690c214f615
Qarton pyproject.toml	ec10aad30fd62b8b98878b0128692a4a8bf44babfc6c52796516a8dda399e3b

The audit did not check byte identity between the bundled publication PDF and the arXiv PDF. The retained manifests record hashes of archives, source trees, and a PDF inspected during capture. The evidence repository retains those manifests, result JSON files, and the capture README, but it does not retain the hashed archives, source trees, or PDF. The revision labels state the intended Git provenance. Neither a recorded hash nor a revision label proves that a remote repository served the inspected bytes at the named revision.

Table 18: Canonical result-file and payload hashes.

Mode	Baseline file / payload	Full-window file / payload
Special space	b14bafb18b8a4bd50d00ab92c07b0ee85b5404c394802b9d9d65a28ce7163d5 / bc71e7d69659f0e97926ffbaa2539032efffa74585ccc7868f6a53f35e8dc3	4881266462b656e8b7655168543c1505f9521fcea964a380f6af2f9d22cd55be / 0885a810a190a4f5660229e50ad7b82893b224fcc950abbbdf6c74422cb45c50
Special gate	6e734152bc395a47df8228e9e546f2afe7ec409e14b383b2e290e1602ab83e08 / 27084e7bf0f01a23766a61caf0e82a854db41b0e6d94e503b9115a8792af984	c921a301c83ae09c9f8a2a4553fd33d5d34f1e3f348bc673e54e450588db7c1 / 5431ac3baf69dc9b67f2e76272641995f4f5fa0117c879adc8aec26c28ae6170

Mode	Baseline file / payload	Full-window file / payload
Generic space	69adac20271d99af5cd075cb4b9f559ee33c8f6c03d30bfdec45ba2608251b4d / ea599e3204152377ce75381e0eb8c903f6f5aebc43ff8c5b724f5fec145c15cd	f160baca524ceab6d84e2391ade356910e11e7e15de104046642c9abc7c0b661 / 371a56232beab9292456c650596ba89ea7ac5bd7fb39c18d2c67b2e3383d19e8
Generic gate	bf1db744ec4f0851ca4b34068c2e8bcc9fbd1e25e89a473781bc3b6293fdd82b / 0ecfa4119f37b5095fc298633ca449038938c238c0ba55d017847fb7948789	bfd1ade981b8bcf36143ca43691c2daaab8221733a457c47eebf062f0e3dbe4 / b549e7a99964c6ff48b75482ea8ba4cb222b189f36a541e7b5c0be4d90cd5a08

The SHA-256 values authenticate retained bytes and the recorded directory walk, not Git-host provenance. The event and payload hashes do not derive Python semantics, circuit cleanup, or physical qubit requirements. Those limits are unchanged by identical repeated outputs.

D.4 Complete resource environment inventory

The schema-v2 manifest records the following installed distributions. The list is an execution identity rather than a claim that every package affects the output. Qarton was installed into the virtual environment from the pinned archive. The isolated commands imported Qarton and companion modules from separate clean source roots placed first on `sys.path`.

Table 19: Installed resource-capture distributions and versions.

Package	Version	Package	Version	Package	Version
antlr4-python3-runtime	4.13.2	numpy	2.5.2	rich	15.0.0
contourpy	1.3.3	openqasm3	1.0.1	scipy	1.18.0
cycler	0.12.1	packaging	26.3	setuptools	84.0.0
fonttools	4.63.0	pillow	12.3.0	six	1.17.0
frozendict	2.4.7	pip	26.2.1	sympy	1.14.0
iniconfig	2.3.0	pluggy	1.6.0	tabulate	0.10.0
kiwisolver	1.5.0	Pygments	2.21.0	tqdm	4.70.0
markdown-it-py	4.2.0	pyparsing	3.3.2	types-tabulate	0.10.0.20260508
matplotlib	3.11.1	pytest	9.1.1	types-tqdm	4.70.0.20260805
mdurl	0.1.2	python-dateutil	2.9.0.post0	typing_extensions	4.16.0
mpmath	1.3.0	qarton	0.2.1	wheel	0.48.0

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