

A Formal and Source Audit of Schrottenloher’s Optimized Point-Addition Circuits for Elliptic-Curve Discrete Logarithms

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Abstract

Schrottenloher proposes low-resource quantum point-addition circuits based on a compressed binary-Euclidean trace, reversed Bézout reconstruction, and approximate modular arithmetic. We audited Section 2, Algorithms 1–11, Figure 1, and Tables 1–3 using Lean 4 proofs, exact arithmetic, and repeated executions of source code pinned to the publication revisions. The audit verifies the compressed trace, reconstruction, exact modular operations, and success-domain behavior of the approximate operations. It also finds a secp256k1 input that needs 405 rather than 402 Euclidean rounds, a shrinking-field failure at round 369, two Table 2 exponents that change by 0.01 when exact source counts replace rounded inputs, and a conditional Table 3 reconstruction that gives 11 percent for an entry printed as 10 percent. The aggregate failure bound remains unsupported: no fixed-denominator theorem proves the 402-round tail, 10,000 successful tests do not establish the stated bound, and no theorem transfers the reference-law arithmetic bounds to Algorithm 1’s call states.

1 Audit scope and evidence

Let p denote the odd prime modulus, n its working bit width, and w the window width. The audit distinguishes value correctness, reversible-circuit action and cleanup, syntax-derived resources, and probability under a stated distribution. A value theorem does not establish cleanup, resources, or probability without a checked connecting theorem, and a component count does not establish a point-addition or whole-ECDLP total. Every numerical probability bound below specializes p to the secp256k1 prime.

The paper’s grouped resource metric adds raw CCX and CCZ gates to charged forward temporary-AND operations. The retained captures contain no raw CCZ gates. Their grouped values therefore equal raw CCX plus forward temporary AND.

The review covers arXiv:2606.02235v1 [3]. Formal evidence comes from VQ revision 00ce72179927 [4]. Section 4 identifies the source revisions used for resource execution. Table 1 summarizes the proved statements, conditional results, counterexamples, and missing evidence for Section 2, Algorithms 1–11, Figure 1, and Tables 1–3.

Table 1: Assessment of Section 2, Algorithms 1–11, Figure 1, and Tables 1–3.

Item	Assessment
§2	A reference theorem checks secp256k1 ECDLP recovery from the ideal Fourier state. The paper supplies no complete generated oracle, induced arithmetic-call law, or proof that one measurement recovers the logarithm with probability greater than one half.
Algorithm 1	The published generic affine path is checked under explicit table and nonexceptional-input premises. Exceptional inputs, concrete point and shift tables, persistent branch-selection wires, composite resources, and the arithmetic input law are omissions.
Algorithm 2	The compressed trace and record circuit are checked. A counterexample rules out termination of every nonzero secp256k1 input within 402 rounds. All such inputs terminate within 511 rounds, and the bound is tight. The 402-round tail probability remains unproved.
Algorithm 3	Compressed Bézout reconstruction and the reverse schedule are checked under terminal-trace and per-step success premises. The checked failure bounds use a separate reference law.
Algorithm 4	The secp256k1 multiplier is checked on its trace, padding, and arithmetic success domain. A 512-round corrected schedule is checked, and a round-369 witness shows that the reported shrinking schedule can clip a live value.
Algorithm 5	The five exact modular-doubling stages have checked action and cleanup. These local theorems do not yield a Schrottenloher Table 1 total.

Item	Assessment
Algorithm 6	Approximate doubling has checked success and failure semantics. At each reverse index, the selected failure event has probability at most $(2^{216} - 1)/p$ under the uniform reference-product law.
Algorithm 7	Pseudo-Mersenne doubling has checked circuit semantics and a per-index failure bound below 2^{-40} under the reference law.
Algorithm 8	The five exact controlled-addition stages have checked action and cleanup. These local theorems do not yield a Schrottenloher Table 1 total.
Algorithm 9	Approximate controlled addition has checked success-domain semantics and a per-index reference-law bound below 2^{-39} .
Algorithm 10	Pseudo-Mersenne controlled addition has checked success-domain semantics. Its arithmetic and cleanup events are each below 2^{-40} per index under the reference law.
Algorithm 11	Equality-aware addition and its three prioritized failure classes are checked. Under the reference law, their union across all 402 reverse indices is below 2^{-29} .
Figure 1	The compressor, reverse action, and valid-block readout are checked. Each block uses exactly 13 gates, including 5 CCX and 6 CNOT gates.
Table 1	Pinned-source executions reproduce the four Schrottenloher grouped gate counts but expose width and full-window surcharge discrepancies. The total-gate rule and captioned failure probability remain unsupported.
Table 2	The factor 28 and source arithmetic are checked. Two gate exponents change by 0.01 under exact source counts. The Babbush formulas reproduce the two comparison rows, but no whole row follows from generated syntax.
Table 3	The audit's inferred class map reproduces 17 of 18 cells. Under that reconstruction, generic modular squaring rounds to 11 percent rather than the printed 10 percent. Schrottenloher's aggregation map is absent.

Figure 1 is the exact interface between the Euclidean trace and its stored record. It packs three valid transition choices from a six-wire block into five retained bits, clears the sixth wire, and reverses to the original block. For the reported 402-round schedule, 134 blocks therefore retain 670 bits. The audit checks the block action, the family action on consecutive blocks, the reverse action, and the exact 13-gate, 5-CCX, 6-CNOT cost per block.

Algorithms 2–4 form one conditional arithmetic chain. Algorithm 2 generates the binary-Euclidean choices, Algorithm 3 reads them backward to reconstruct Bézout coefficients, and Algorithm 4 uses the forward trace and reverse reconstruction to multiply in place. The checked reconstruction invariant relates the current coefficients to the exact remainder pair, including a final compressed block containing one or two choices. Multiplication follows when the fixed trace terminates and every selected approximate reconstruction step satisfies its success predicate. Failure of either premise lies outside that theorem.

Algorithms 5 and 8 admit stronger local conclusions. Their five published stages have checked whole-register action, preserve the stated inputs, and clear their ancillas for exact modular doubling and exact controlled addition. Algorithms 6, 7, and 9–11 instead have total failure classifications and circuit theorems restricted to explicit success domains. Those distinctions permit finite probability bounds for named events without treating an out-of-domain input as proof that the generated circuit returns an incorrect value.

2 The 402-round and probability claims

Section 3.1 models the binary-Euclidean iteration count with mean $1.413n$ and an experimentally observed standard deviation near $0.6\sqrt{n}$. The companion evaluates

$$3 \left\lceil \frac{1.413n + 2.4\sqrt{n}}{3} \right\rceil,$$

which gives 402 at $n = 256$ after rounding the schedule to a multiple of three. The cited binary-Euclid analysis calls the product-shrink and normal-tail statements heuristics [2]. Neither source proves a fixed-denominator bound for the number of secp256k1 residues whose traces exceed 402 rounds.

For a live binary-Euclidean transition from (u, v) to (u', v') , the checked proof establishes $2u'v' \leq uv$. It follows that every nonzero secp256k1 residue terminates within 511 rounds, and the input 2^{255} first terminates at round 511, so the bound is tight. A separate checked residue remains at $(1, 7)$ after round 402 and reaches the terminal state at round 405. A fixed 512-round reversible multiplier therefore supplies a deterministic correction with checked multiplication semantics and cleanup.

The companion's shrinking active fields introduce a separate failure. At round 369, one checked input reaches the state

$$(6033964351, 44029).$$

The retained comparison agrees with the full comparison, but clipping changes the next exact state from

$$(44029, 3016960161) \quad \text{to} \quad (44029, 869476513).$$

This witness isolates a field-size failure and gives the shrinking Algorithm 2 trace-model value-failure probability a lower bound of $1/(p-1)$ under uniform nonzero inputs.

Schrottenloher’s Table 1 associates failure probability at most $2^{-13.3}$ with 10,000 random tests that reported no failures. If independent trials had true failure probability $1/10,000$, the probability of observing zero failures would be $(9999/10000)^{10000}$, which lies between one third and one half. The one-sided 95-percent Clopper–Pearson upper endpoint after zero failures is $1 - 0.05^{1/10000} \approx 2.995 \cdot 10^{-4}$, above both $1/10100$ and $2^{-13.3}$. Under an IID-uniform model for accepted nonzero residues, a separate million-residue sample found 19 traces longer than 402 and has a one-sided 95-percent upper endpoint near $2.7879 \cdot 10^{-5}$, below $1/10100$. The retained record has no durable raw transcript. Lean checks the listed failures and checkpoints and exact binomial formulas, but it does not check the IID premise, transcript provenance, million-trial numerical enclosure, or Clopper–Pearson bracket. The sample supplies statistical evidence for the termination event but proves neither the finite failure fraction nor the complete-circuit bound.

The checked arithmetic probabilities fix p to the secp256k1 prime and use a uniform reference product: the Euclidean input is uniform on $\{1, \dots, p-1\}$ and the reconstruction coefficient is independently uniform modulo p . At each reverse index, the selected Algorithm 6 event is at most $(2^{216} - 1)/p$, the combined Algorithm 7 event is below 2^{-40} , and the combined Algorithm 9 arithmetic-and-cleanup event is below 2^{-39} . For Algorithm 10, addition arithmetic after successful doubling and cleanup after prior success are each below 2^{-40} per reverse index. The three Algorithm 11 failure classes have union below 2^{-29} across all 402 reverse indices. The uniform law on the two 256-bit basis-label registers cannot push forward exactly to the reference product by cardinality, and the paper supplies neither a call-state distribution nor an event-domination theorem. The bounds therefore do not prove Schrottenloher’s Table 1 probability.

3 Resource tables

The resource audit executed the four publication modes twice with the two-entry baseline lookup and twice with a direct 65,536-row window. The publication command fixes the two-row table, so the full-window runner directly instantiated the pinned point-addition class with all 65,536 rows while retaining the four mode choices. The paired outputs are byte-identical, all runs report zero raw CCZ gates, and Table 2 records the measured widths and grouped counts in this order: secp256k1 space, secp256k1 gate, generic-prime space, and generic-prime gate. These are pinned-source execution results rather than syntax-derived theorems about the publication’s unrecorded environment.

The static source equations and captured executions agree on all four grouped baseline totals and all four full-window totals. The captured root events satisfy Qarton’s peak-width recurrence and account for each measured width. The audit does not derive those liveness traces from source syntax.

Table 2: Publication-pinned baseline and full-window executions.

Mode	Baseline		Full window	
	Qubits	Grouped	Qubits	Grouped
secp256k1, space	1,192	2,385,517	1,207	2,586,927
secp256k1, gate	1,445	1,865,521	1,460	2,066,931
any prime, space	1,191	3,608,043	1,206	3,809,453
any prime, gate	1,446	2,803,519	1,461	3,004,929

The paper’s baseline widths are 1,192, 1,446, 1,192, and 1,446 in the same row order, so the special-prime gate and generic-prime space entries each exceed the capture by one qubit. The full source adds 15 qubits because its 16-bit address replaces the one-bit baseline address. It also adds 201,410 grouped units in every mode: 196,602 for three forward lookups, 4,584 for three measured unlookups, and 224 for four nonzero tests. The paper instead adds 16 qubits and $3 \cdot 2^{16} = 196,608$ grouped units, omitting 4,802 units from the source-derived increment.

Two 256-bit scalars at window width 16 give 32 window ordinals. The paper replaces the first point addition with a direct lookup and removes the last three point additions at the cost of classical postprocessing, leaving 28 retained point additions. Its printed exponents 26.11, 25.78, 26.66, and 26.33 follow Schrottenloher’s rounded Table 1 inputs. Exact full-window source counts instead give 26.11, 25.79, 26.67, and 26.33, changing the special-prime gate and generic-prime space entries by 0.01. The printed widths 1,208, 1,462, 1,208, and 1,462 exceed the captures by 1, 2, 2, and 1 qubits. These core formula estimates omit the initial direct lookup.

Schrottenloher’s Tables 1 and 2 reproduce two rows from Babbush et al. [1]. Appendix A of that paper gives qubit bounds of 1,175 and 1,425 and upper bounds of 2,700,000 and 2,100,000 on the reported average executed CCX-plus-CCZ counts over 9,024 hash-derived pseudorandom tests. Substitution into equations (A1)–(A3), with $w = 16$ and 28 additions, gives the core bounds

$$28(2,700,000 + 3 \cdot 2^{16}) = 81,105,024, \quad 28(2,100,000 + 3 \cdot 2^{16}) = 64,305,024.$$

The point-addition bounds round to $2^{21.36}$ and $2^{21.00}$, matching Schrottenloher’s Table 1 entries. The Appendix A qubit-bound formula gives bounds $1175 + 16 = 1191$ and $1425 + 16 = 1441$, and the gate bounds round to $2^{26.27}$ and $2^{25.94}$, matching Schrottenloher’s Table 2 entries. Babbush et al. withhold the circuits and publish a zero-knowledge attestation of the 9,024-case computation. Appendix A gives no numerical bounds for the initial direct lookup, lookup uncomputation, or recycled-QFT work. The formulas therefore verify the displayed core bounds, rather than exact whole-circuit totals or Schrottenloher’s Table 1 failure caption.

Schrottenloher’s Table 3 reports component percentages for the two space-optimized modes. The source reporter recursively aggregates costs by Python class, but parent and child classes overlap and the archive does not retain Schrottenloher’s class map or aggregation order. Under the audit’s inferred class map and rounding rule, rounding each class proportion before summing reproduces all nine special-prime cells and eight generic-prime cells. The remaining generic modular-squaring ratio 380314/3608043 is approximately 10.540728 percent, which rounds to 11 percent rather than the printed 10 percent.

The retained sources do not define the total-gate column. They supply neither the included operation classes nor a rounding rule, and adding every basic reporter entry does not reproduce a defined quantity. The audit records the grouped CCX/CCZ/AND figures, widths, and component reconstruction while leaving the total-gate column unsupported. Deriving a whole-ECDLP resource row would also require generated syntax for the complete two-scalar ECDLP oracle and a proof of its peak liveness.

4 Reproducibility and remaining questions

The publication resource bundle is companion revision 3b60d5050b18, which pins Qarton revision 7506faba2147. The 402-round source audit uses companion revision d8d2473cb43c. Its Euclidean schedule agrees with the publication bundle. We did not establish byte identity between the bundled PDF and arXiv v1. The artifact retains the archive hashes and capture manifests.

The repeated resource suites used uv-managed CPython 3.12.12 and Qarton 0.2.1. The publication supplies no dependency lockfile, so the captures establish the pinned sources under the recorded dependency versions rather than byte identity with Schrottenloher’s execution. The Lean 4 evidence is retained at 00ce72179927. The final `lake -rehash -reconfigure build Tests` aggregate completed 4,596 of 4,596 build jobs with status zero under Lean 4.32.0. The artifact records the formal statements, source transcriptions, capture manifests, and exact counterexample residues.

Three questions determine whether the unresolved paper claims can be closed. The probability result needs a fixed-denominator 402-round theorem with explicit `secp256k1` constants, followed by a derivation of the arithmetic call-state law or event-specific domination from Algorithm 1. The resource tables need Schrottenloher’s total-gate rule, Table 3 class map and rounding order, and a dependency-locked execution record or a syntax-derived liveness proof. Circuit-level verification of the Babbush rows requires the withheld circuits. Independent verification of the published Zenodo program, public outputs, verification keys, and Groth16 proofs could establish their stated resource-and-test computation, but not universal point-addition correctness.

The checked arithmetic and circuit results establish the paper’s local algorithmic core on stated domains. A deterministic multiplier needs the checked 512-round schedule and fields wide enough to retain live remainders. A probabilistic 402-round construction needs the missing tail and input-law bounds. The source audit identifies the resource-table discrepancies, while the whole-resource and aggregate-probability claims remain unsupported by the available record.

References

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