

VQ System Design: Connecting Quantum Programs, Proofs, and Resource Reports

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Abstract

Quantum-algorithm evidence often assigns functional correctness, success probability, and resource cost to artifacts produced by separate tools. **VQ** addresses that provenance problem in Lean 4 by defining circuits and hybrid programs as authoritative syntax, proving behavior over that syntax, and deriving logical resource reports from it. This note specifies **VQ**'s design goals, architectural layers, trust boundary, symbolic strategy for circuit families, and resource and probability endpoints. It also records the current limits: terminal hybrid-program distributions and most circuit-to-complex approximation results remain test-local, reports stop at logical gates and dependency depth, and the Lean-native VECDSA game lacks an external hostile-submission adapter.

1 Design goals fix the evidence object

VQ connects one generated Lean artifact to its semantic theorem and logical resource report, or connects distinct representations through proved translations. Lean serves as the generator language, proof language, and authoritative program representation, so a resource figure cannot refer to a different circuit unless a theorem relates the two. The earlier **VQ** paper develops the exact semantics, verified corpus, and research program in detail [4]. This note isolates the design and its current evidence boundaries.

The design separates reusable verification from challenge-specific policy. The general library defines circuit and program syntax, semantics, specifications, approximation relations, and resource functions, while a game selects an operation, accepted gate basis, score, and candidate package. This separation permits a competition to change its score without changing the meaning of a circuit, and it permits researchers to use the verification library without adopting the VECDSA game [3].

2 Architectural layers preserve distinct semantics

Table 1 records the implemented layer boundaries. The unitary **Circuit** type remains separate from **Program**, whose measurement, reset, storage, and branch operations denote lists of subnormalized branches. This split follows the semantic distinction between a linear state map and a hybrid execution record, a distinction also present in verified quantum languages such as SQIR and CoqQ [2, 5].

Exact amplitudes use the dyadic cyclotomic ring $\mathbb{Z}[\zeta][1/2]$ at a selected power-of-two phase level. The finite representation supports decidable equality for exact gate identities, while **VQBridge** proves its embedding into the complex numbers and connects gate actions to standard complex matrices. Continuous angles require an approximation statement, so the production **VQ.Approx** layer defines state, operator, and projective-state error relations rather than assigning an exact scalar to an unsupported rotation [4, 3].

Measured components use a stronger endpoint than agreement on basis outputs. **RealisesAt** chooses an output map and one amplitude for each measurement record before quantifying over the basis input, which prevents the record amplitude from depending on protected quantum data. Its linearity theorems then transport the component action to finite superpositions, as exercised by the parameterized measured-adder and measured-QROM families in Examples 21 and 23 [3].

Layer	Responsibility	Checked source endpoint
Circuit syntax	Finite gate lists, well-formedness, composition, coupling, and logical counts and depths	<code>VQ.Circuit</code> , <code>Circuit.gateCount</code> , <code>Circuit.depth</code>
Program syntax	Measurement, reset, immutable input, mutable storage, feed-forward, and branch-cost ranges	<code>VQ.Program</code> , <code>Program.depthRange</code>
Exact semantics	Dyadic cyclotomic vectors, gate and circuit action, branch execution, full outcomes, and selected-wire marginals	<code>Semantics.run</code> , <code>Semantics.runProgram</code> , <code>probAt</code> , <code>probMarginal</code>
Reversible arithmetic	Basis-index permutations and compilation to the ordinary circuit syntax	<code>Reversible.act</code> , <code>run_compile_basis</code> , <code>denote_compile_eq</code>
Specifications	Register relations and coherent realization of measured components	<code>RegSpec</code> , <code>RealisesAt</code> , <code>ImplementsU</code>
Certificates	Compositional bounds for reversible lists, circuits, and embedded programs	<code>ReversibleCertificate</code> , <code>CircuitCertificate</code> , <code>ProgramCertificate</code>
Game and judge	A dependent program, proof, admission, and score package, older manifest obligation construction and audits	<code>VQ.Game.VECDSA.Candidate</code> , <code>Judge.Obligation</code> , <code>Judge.Audit</code>

Table 1: Implemented layers and representative checked declarations in artifact revision 369a5fc.

3 Circuit families use symbolic proof interfaces

VQ uses direct evaluation for small finite circuits and symbolic theorems for families whose state spaces are too large to materialize. A family is an ordinary Lean function that emits syntax from parameters, accompanied by well-formedness, semantic, cleanup, and resource theorems over the same parameters. Cryptographic-width arithmetic proofs reason about basis-index permutations, register equations, finite sums, and recurrences instead of enumerating 2^n state entries [4, 3].

The proof interface follows the circuit structure. Reversible arithmetic uses `Reversible.act` and the verified compiler, diagonal operations use phase functions, Fourier operations use finite character sums, and hybrid components use `RealisesAt` or `ImplementsU`. Resource certificates compose over list append, repetition, compilation, and embedding, so a symbolic bound refers to the generated syntax rather than to a copied estimate [3].

The VECDSA package demonstrates the binding at a closed cryptographic width. `Candidate` stores one `Program`, a proof of `Correct program`, and a proof of gate-basis admission, while `Candidate.report` computes every resource field from that stored program. The baseline’s `program_correct`, `program_admissible`, and `candidate` declarations therefore share one program value, and its report theorems bound the syntax that the correctness theorem names [4, 3].

4 Resource and probability endpoints state different claims

Circuit resources are exact functions of the gate list, including gate-class counts, used wires, and greedy per-wire logical depth. Program resources use a closed interval: branch composition selects the minimum and maximum syntactic arm costs, and straight-line code has equal endpoints. The VECDSA `ResourceReport` adds storage use, measurements, resets, feed-forward and classical depths, a seven-T-per-CCZ expansion bound, and the game score, all computed from program syntax [3].

The production circuit semantics supplies complete-basis probabilities through `probAt`, exact certainty and impossibility predicates, and selected-wire marginals through `probMarginal`. Rational lower and upper bounds are decidable at phase level three, where the real subring has the implemented order. Program semantics assigns each branch the squared norm of its subnormalized state and proves total-probability preservation, while the general terminal event interface remains in `Tests.ProgramObservation` rather than the public VQ namespace [3].

These endpoints answer separate questions. A component theorem establishes a coherent transformation that a later circuit may reuse, while a terminal probability theorem establishes the distribution of an observation

that the algorithm exposes. A resource theorem establishes logical syntax and dependency costs, and it makes no claim about routing, error correction, factories, physical latency, or expected branch cost.

5 The trust boundary depends on the verification path

The scientific result consists of the Lean source, pinned Lean 4.32.0 toolchain and dependencies, kernel-checked declarations, and disclosed axiom dependencies. Permanent axiom guards for central results admit `propext`, `Classical.choice`, and `Quot.sound`. The source uses no separate resource simulator to populate a candidate report. Lean’s small kernel checks proof terms, while Mathlib and the VQ definitions remain part of the statement and proof dependency graph [1, 3].

The older manifest judge adds controls for an adversarial submission. Trusted code constructs each required proposition as a Lean expression, compares it with the submitted declaration type, walks the declaration graph for forbidden axioms and unsafe code, and invokes `leanchecker` to replay kernel checks. The project does not claim that `leanchecker` independently implements Lean’s kernel, and building author code still requires process isolation for machine security [4, 3].

The Lean-native VECDSA game currently follows a different operational path. An in-tree build checks the dependent candidate, negative type tests, and axiom guards, but no external adapter constructs or audits that candidate from an isolated submission package. The baseline consequently constitutes checked scientific evidence, while an adversarial external VECDSA verdict remains unavailable [3].

6 Current limits identify the next interfaces

Three general interfaces remain outside the production library: a terminal hybrid-program distribution, a circuit-to-complex approximation bridge, and semantic combinators for tensoring and controlling arbitrary circuits. Tests contain partial and specialized results in these directions, including terminal events, QFT and phase-estimation families, approximation transport, amplitude amplification, repetition, and an ECDLP reference circuit. Their location under `Tests` records that the kernel checks them while their production API and external claim format remain unsettled [3].

Direct circuit evaluation grows with the full basis, and program evaluation also branches on measurements. Large proofs therefore require algorithm-specific symbolic arguments, most of which use handwritten induction, arithmetic invariants, finite-sum identities, and explicit resource recurrences. The repository has no general path-sum normalizer, verified optimizer, or automated hybrid-program invariant engine [3].

Resource reports end at logical gate classes and logical dependency depth. Program ranges do not prove branch reachability and do not weight costs by branch probability, while coupling checks do not supply a verified router. Code selection, magic-state factories, physical error rates, and timing models therefore lie outside every current certified cost [3].

References

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